Surds – GCSE Maths

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1. Introduction

- **Surds** are numbers written in square root form that produce irrational numbers.
- They are used when we work with roots in their exact form.
- In many maths problems, we need to keep numbers in root form instead of changing them into decimals.

Example:

$\sqrt{5}$ is a surd because it is an irrational number and cannot be written as a simple fraction.

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2.What are Rational and Irrational Numbers?

In mathematics, numbers are divided into rational and irrational numbers.

- **Rational numbers:**

Numbers that can be written as a simple fraction $\frac{a}{b}$ (where b ≠ 0) or as a whole number are called **rational numbers**.

Example:

- $\frac{3}{4} = 0.75$
- $0.333... = \frac{1}{3}$
- 100, 0 etc.
- -3, -2 etc.

- **Irrational numbers:**

Numbers that cannot be written as a simple fraction $\frac{a}{b}$ (where b ≠ 0) are called **irrational numbers.**

Example:

- $\sqrt{5}$
- $\sqrt{2} \approx 1.4142...$
- $\pi \approx 3.1415...$
- $\sqrt{3} \approx 1.7320...$

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3.What are Surds?

- A surd is a root that cannot be simplified into a whole number.
- Surds usually occur when we take the root of numbers that are not perfect powers.
- These roots can be square roots, cube roots, or other roots. Instead of writing them as long decimal numbers, we keep them in root form to show the exact value.

Example:

$\sqrt[3]{2}$

→ This is also a surd because it cannot be written as a whole number.

$\sqrt{5}$

→ This is a surd because 5 is not a perfect square.

$\sqrt{16} = 4$

→ This is not a surd because 16 is a perfect square and the root gives a whole number.

$\sqrt{9} = 3$

→ This is not a surd because 9 is a perfect square and the root gives a whole number.

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4.How to simplify Surds?

Simplifying surds means taking out perfect square factors from inside the square root.

Steps to Simplify Surds:

Step#1: Break the number into factors using a factor tree.

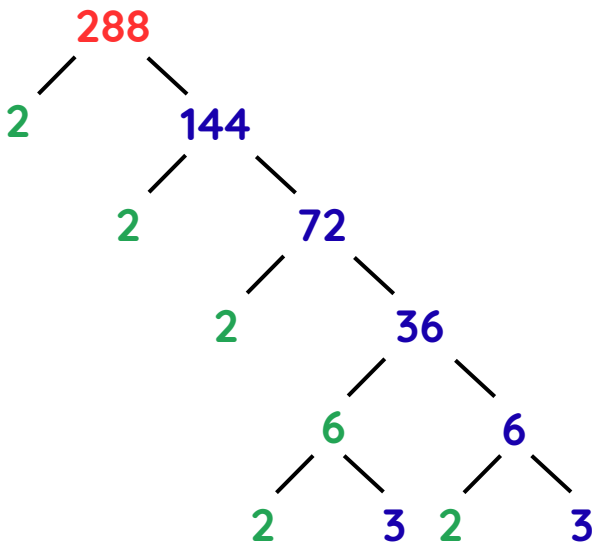
Step#2: Group the same numbers into pairs.

Step#3: Take each pair outside the root and leave any leftover inside.

Example 1: Simplify $\sqrt{288}$

Solution:

Step#1: Break the number into factors using a factor tree.



So,

$$288 = 2 \times 2 \times 2 \times 2 \times 2 \times 3 \times 3$$

Step#2: Group the same numbers into pairs.

$$(2 \times 2)(2 \times 2)(3 \times 3) \times 2$$

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Step#3: Use root and take pairs outside

$$\sqrt{288} = \sqrt{(2 \times 2)(2 \times 2)(3 \times 3) \times 2}$$

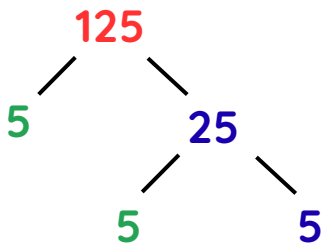
$$\sqrt{288} = 2 \times 2 \times 3\sqrt{2}$$

$$\sqrt{288} = 12\sqrt{2}$$

Example 2: Simplify $5\sqrt{125}$

Solution:

Step#1: Break the number into factors using a factor tree.



So,

$$125 = 5 \times 5 \times 5$$

Step#2: Group into pairs

$$(5 \times 5) \times 5$$

Step#3: Use root and take pairs outside

$$5\sqrt{125} = 5\sqrt{(5 \times 5) \times 5}$$

$$5\sqrt{125} = 5 \times 5\sqrt{5}$$

$$5\sqrt{125} = 25\sqrt{5}$$

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5.How to Add and Subtract Surds?

Surds can be added or subtracted only when they have the same root.

Steps to Add and Subtract Surds:

Step#1: Simplify the surds and check they have the same root

Step#2: Add or subtract the coefficients

Step#3: Keep the root the same

Example 1: Simplify $\sqrt{12} + 3\sqrt{27}$

Solution:

Step#1: Simplify the surds and check they have the same root

$$\sqrt{12} = \sqrt{4 \times 3} = 2\sqrt{3}$$

$$\sqrt{27} = \sqrt{9 \times 3} = 3\sqrt{3}$$

Now,

$$\sqrt{12} + 3\sqrt{27} = 2\sqrt{3} + 3(3\sqrt{3})$$

$$= 2\sqrt{3} + 9\sqrt{3}$$

Both surds now have the same root.

Step#2: Add the coefficients

$$2 + 9 = 11$$

Step#3: Keep the root the same

Final Answer:

$$11\sqrt{3}$$

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Example 2: Simplify $5\sqrt{50} - 2\sqrt{8}$

Solution:

Step#1: Simplify the surds and check they have the same root

$$\sqrt{50} = \sqrt{25 \times 2} = 5\sqrt{2}$$

$$\sqrt{8} = \sqrt{4 \times 2} = 2\sqrt{2}$$

Now,

$$5\sqrt{50} - 2\sqrt{8} = 5(5\sqrt{2}) - 2(2\sqrt{2})$$

$$= 25\sqrt{2} - 4\sqrt{2}$$

Both surds now have the same root.

Step#2: Add the coefficients

$$25 - 4 = 21$$

Step#3: Keep the root the same

Final Answer:

$$21\sqrt{2}$$

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6.How to Multiply and Divide Surds?

Surds can be multiplied or divided by multiplying or dividing the numbers outside the roots and the numbers inside the roots, while keeping the answer in root form.

Steps to Add or Subtract Surds:

Step#1: Multiply or divide the coefficients (the numbers outside the roots)

Step#2: Multiply or divide the numbers inside the roots

Step#3: Simplify the surd if possible

Example 1: Simplify $3\sqrt{2} \times 4\sqrt{5}$

Solution:

Step#1: Multiply the coefficients

$$3 \times 4 = 12$$

Step#2: Multiply the numbers inside the roots

$$\sqrt{2} \times \sqrt{5} = \sqrt{10}$$

Step#3: Write the simplified answer

$$12\sqrt{10}$$

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Example 2: Simplify $\frac{6\sqrt{45}}{3\sqrt{5}}$

Solution:

Step#1: Simplify the surds and divide the coefficients

$$\frac{6}{3} = 2$$

$$\sqrt{45} = \sqrt{9 \times 5} = 3\sqrt{5}$$

Now,

$$\frac{6\sqrt{45}}{3\sqrt{5}} = \frac{6(3\sqrt{5})}{3\sqrt{5}}$$

$$= \frac{18\sqrt{5}}{3\sqrt{5}}$$

Step#2: Divide the numbers inside the roots

$$\frac{\sqrt{5}}{\sqrt{5}} = 1$$

Step#3: Simplify the answer

$$\frac{18}{3} = 6$$

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7.How to rationalise the denominator?

Rationalising the denominator means removing the surd from the denominator of a fraction so that the denominator becomes a rational number.

Steps to rationalise the denominator:

Step#1: Identify the surd in the denominator

Step#2: Multiply the numerator and denominator by the same root to remove the surd.

Step#3: Simplify the expression if possible.

Example 1: Simplify $\frac{3}{\sqrt{5}}$

Solution:

Step#1: Identify the surd in the denominator

The denominator contains the surd: $\sqrt{5}$

Step#2: Multiply the numerator and denominator by

$$= \sqrt{5}$$

$$\frac{3}{\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}}$$

Step#3: Simplify the expression

$$\frac{3\sqrt{5}}{\sqrt{5} \times \sqrt{5}} = \frac{3\sqrt{5}}{5}$$

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Example 2: Rationalise the denominator of $\frac{5}{2\sqrt{3}}$

Solution:

Step#1: Identify the surd in the denominator

The denominator contains the surd: $2\sqrt{3}$

Step#2: Multiply the numerator and denominator by

$$= \sqrt{3}$$

$$\frac{5}{2\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$$

Step#3: Simplify the expression

$$\frac{5\sqrt{3}}{2\sqrt{3} \times \sqrt{3}} = \frac{5\sqrt{3}}{2 \times 3}$$

$$= \frac{5\sqrt{3}}{6}$$

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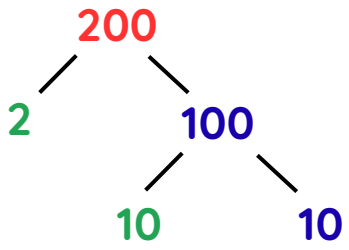
8.Solved Examples

Problem 1: Solve $\sqrt{200} - \sqrt{72}$

Solution:

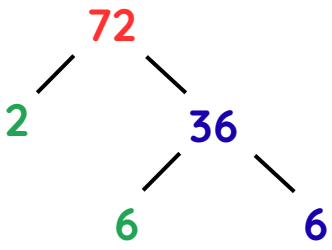
Step#1: Simplify the surds

- For $\sqrt{200}$:



$$\sqrt{200} = \sqrt{(10 \times 10) \times 2} = 10\sqrt{2}$$

- For $\sqrt{72}$:



$$\sqrt{72} = \sqrt{(6 \times 6) \times 2} = 6\sqrt{2}$$

Step#2: Subtract the coefficients

$$10\sqrt{2} - 6\sqrt{2} = (10 - 6)\sqrt{2}$$

Step#3: Keep the root same

$$= 4\sqrt{2}$$

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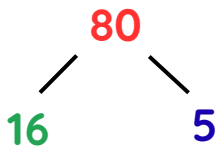
Problem 2: Simplify $\sqrt{80} - \frac{10}{\sqrt{5}}$

Solution:

Step#1: Simplify the surds



$$\sqrt{80} = \sqrt{16 \times 5} = 4\sqrt{5}$$

Step#2: Rationalise the denominator

$$\frac{10}{\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}} = \frac{10\sqrt{5}}{5}$$

$$= 2\sqrt{5}$$

Step#3: Subtract the surds

$$4\sqrt{5} - 2\sqrt{5} = (4 - 2)\sqrt{5}$$

$$= 2\sqrt{5}$$

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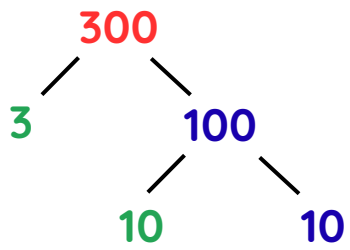
Problem 3: Solve $\sqrt{300} - 2\sqrt{x} = 4\sqrt{3}$

Solution:

Step#1: Break the number into factors using a factor tree.



$$\sqrt{300} = \sqrt{100 \times 3} = 10\sqrt{3}$$

Step#2: Substitute and simplify equation

$$10\sqrt{3} - 2\sqrt{x} = 4\sqrt{3}$$

Step#3: Rearrange and solve

$$10\sqrt{3} - 4\sqrt{3} = 2\sqrt{x}$$

$$6\sqrt{3} = 2\sqrt{x}$$

Divide both sides by 2:

$$3\sqrt{3} = \sqrt{x}$$

Step#4: Square both sides

$$(3\sqrt{3})^2 = x$$

$$= 9 \times 3 = 27$$

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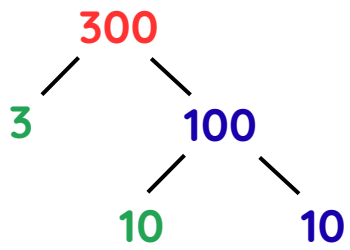

$$\sqrt{300} = \sqrt{100 \times 3} = 10\sqrt{3}$$

Step#2: Substitute and simplify equation

$$10\sqrt{3} - 2\sqrt{x} = 4\sqrt{3}$$

Step#3: Rearrange and solve

$$10\sqrt{3} - 4\sqrt{3} = 2\sqrt{x}$$

$$6\sqrt{3} = 2\sqrt{x}$$

Divide both sides by 2:

$$3\sqrt{3} = \sqrt{x}$$

Step#4: Square both sides

$$(3\sqrt{3})^2 = x$$

$$= 9 \times 3 = 27$$

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Problem 3: Show that $\frac{6}{2-\sqrt{3}}$ can be written in the form $a + b\sqrt{3}$

Solution:

Step#1: Multiply by conjugate

$$\frac{6}{2-\sqrt{3}} \times \frac{2+\sqrt{3}}{2+\sqrt{3}}$$

Step#2: Expand numerator and denominator

- Numerator:

$$6(2+\sqrt{3}) = 12 + 6\sqrt{3}$$

- Denominator:

$$(2-\sqrt{3})(2+\sqrt{3}) = 4 - 3 = 1$$

Step#3: Write final answer

$$\frac{12+6\sqrt{3}}{1} = 12 + 6\sqrt{3}$$

So,

- **a** = 12
- **b** = 6