

HCF AND LCM – GCSE MATHS

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1. Introduction

In mathematics, two very **useful concepts** when working with numbers are HCF (Highest Common Factor) and LCM (Least Common Multiple). They help us solve problems related to **divisibility, fractions, ratios, and more.**

- The **Highest Common Factor (HCF)** is the **greatest number** that divides two or more integers exactly, without leaving a remainder. It is used to find common factors and simplify numerical expressions.
- The **Least Common Multiple (LCM)** is the **smallest number** that is a multiple of two or more integers. It helps in solving problems involving common time intervals, synchronization, and operations with fractions.



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2. What do you mean by Prime and Common factors?

FACTORS:

A Factor of a number is any whole number that divides it exactly without leaving a remainder.

Example: Find the factors of 20

Solution:

A factor is any number that divides 20 exactly without leaving remainder.

The factors of 20 are: 1, 2, 4, 5, 10, 20

$$1 \times 20 = 20$$

$$2 \times 10 = 20$$

$$4 \times 5 = 20$$

So, all these numbers are factors of 20 because they divide 20 completely.

Now, let us understand Two different types of Comon Factors:

- **Prime Factors:** Prime factors are the building blocks of a number — they are the prime numbers that you multiply together to get that number.
- **Common Factors:** Common factors are the factors that two or more numbers share — in other words, the numbers that can divide all of them without leaving a remainder.

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PRIME FACTORS

Prime factors are the factors of a number that are prime numbers. In other words, they are prime numbers that multiply together to give the original number.

A prime number is a number that has only two factors: 1 and itself (like 2, 3, 5, 7, 11...).

Example:

Let's take the number 36

- The numbers we can multiply to get 18 are

$$2 \times 3 \times 3 \times 2$$

- So, the prime factors of 18 are 2 and 3

We can also write it as:

$$18 = 2 \times 3 \times 3 \times 2 \text{ (only prime numbers)}$$

COMMON FACTORS

Common factors are the numbers that are factors of two or more numbers at the same time.

Example:

Let's take 12 and 16

- Factors of 12 = 1, 2, 3, 4, 6, 12
- Factors of 16 = 1, 2, 4, 8, 16
- The numbers they both have are: 1, 2, and 4

So, the common factors of 12 and 16 are 1, 2, and 4

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3. How to calculate HCF and LCM of a number.

HCF OF A NUMBER

- HCF stands for Highest Common Factor.
- It is the largest number that can exactly divide two or more given numbers without leaving a remainder.

How to Find HCF

The most famous method to find HCF is Prime Factorization method.

Let us perform Prime Factorization method:

For Example: Find the HCF of 220 and 180

Solution:

Prime Factors of 220:

$$220 = 2 \times 2 \times 5 \times 11 = 2^2 \times 5 \times 11$$

◆ Prime Factors of 180:

$$180 = 2 \times 2 \times 3 \times 3 \times 5 = 2^2 \times 3^2 \times 5$$

◆ Now take the common prime factors with the lowest powers:

- 2^2 (common in both)
- 5 (common in both)

So,

$$\text{HCF} = 2^2 \times 5 = 4 \times 5 = 20$$

| | | | |
|---|-----|----|-----|
| 2 | 180 | 2 | 220 |
| 2 | 90 | 2 | 110 |
| 3 | 45 | 5 | 55 |
| 3 | 15 | 11 | 11 |
| 5 | 5 | | 1 |
| | 1 | | |

Hence, HCF of 220 and 180 is 20

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LCM OF A NUMBER

- LCM stands for Least Common Multiple.
- It is the smallest number that is a multiple of two or more numbers.

How to Find LCM

The method that is commonly used to find LCM is Prime Factorization method.

Let us perform Prime Factorization method:

For Example: Find the LCM of 108 and 120

Solution:

- $220 = 2^2 \times 5 \times 11$
- $180 = 2^2 \times 3^2 \times 5$

To find LCM, take all prime factors from both numbers, using the highest power of each:

- 2^2 (from both)
- 3^2 (from 180)
- 5 (common)
- 11 (from 220)

Now multiply:

$$\text{LCM} = 2^2 \times 3^2 \times 5 \times 11$$

$$= 4 \times 9 \times 5 \times 11$$

$$= 36 \times 5 = 180$$

$$180 \times 11 = 1980$$

| | | | |
|-----|---|-----|---|
| 108 | 2 | 120 | 2 |
| 54 | 2 | 60 | 2 |
| 27 | 3 | 30 | 2 |
| 9 | 3 | 15 | 3 |
| 3 | 3 | 5 | 5 |
| 1 | | 1 | |

Hence, LCM of 220 and 180 = 1980

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4. Three Additional Solved Examples

Example 1: Three ropes are 60 cm, 84 cm, and 108 cm long. What is the longest piece length that can be used to cut all of them into equal pieces without any rope left over?

Solution: This is a HCF problem, because we are dividing into equal parts.

Find HCF of 60, 84, and 108.

- $60 = 2^2 \times 3 \times 5$
- $84 = 2^2 \times 3 \times 7$
- $108 = 2^2 \times 3^3$

Common = $2^2 \times 3 = 12$

Hence, the longest piece that can be used is of **12 cm**.

Example 2: (a) Is there a formula connecting HCF and LCM?

(b) If the HCF of two numbers is 1, what does that mean?

Solution:

(a) Yes! For any two positive integers a and b:

$$\text{HCF}(a, b) \times \text{LCM}(a, b) = a \times b.$$

This means the product of HCF and LCM of two numbers is equal to the product of the numbers themselves.

(b) It means the numbers are co-prime (No common factors)

In such cases:

$$\text{HCF} = 1 \Rightarrow \text{LCM} = a \times b$$

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Example 3: Find the HCF and LCM of 15 and 25

Solution:

Using Prime Factorization,

$$15 = 3 \times 5$$

$$25 = 5 \times 5 = 5^2$$

HCF:

Common prime factor = 5

$$\text{HCF} = 5$$

Therefore, HCF of 15 and 25 comes out to be 5.

LCM:

Take all prime factors with highest powers:

$$3, 5^2 \rightarrow 3 \times 25$$

$$= 75$$

Therefore, LCM of 15 and 25 comes out to be 75.