

VECTORS – GCSE MATHS

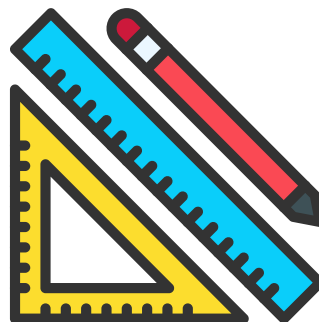
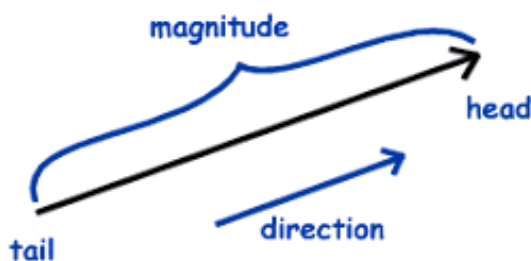
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1. Introduction

Vectors and geometric proofs play a vital role in understanding shapes, directions, and logical reasoning in mathematics.

- A vector is a quantity that has **both magnitude and direction**, often used to describe motion or force in space.
- In contrast, geometric proofs involve **step-by-step reasoning** to show why a geometric statement is true, using definitions, theorems, and logic.



NOTE: Together, these concepts help build a deeper understanding of spatial relationships and mathematical structure.

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2. Terms related to Vectors.

Vector Notation

Vector notation is the way we represent vectors in writing — showing both their **magnitude and direction**.

There are three main ways to write a vector:

1. Arrow Notation

AB This means a vector pointing from point A to point B. The arrow shows direction.

2. Component (Coordinate) Form

$$\vec{A} = \langle x, y \rangle \text{ (in 2D)}$$

This shows the **horizontal**, **vertical**, and (if in 3D) **depth components** of a vector.

3. Unit Vector Notation

- $$\vec{A} = xi + yj \text{ (in 2D)}$$

i, j, k are unit vectors along the x, y, and z axes.

- This form helps break the vector into direction-wise components.

Example:

If a vector moves 3 units right and 4 units up:

- Arrow form: AB
- Component form: $\langle 3, 4 \rangle$
- Unit vector form: $3\hat{i} + 4\hat{j}$

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Vector Length (Magnitude of a Vector)

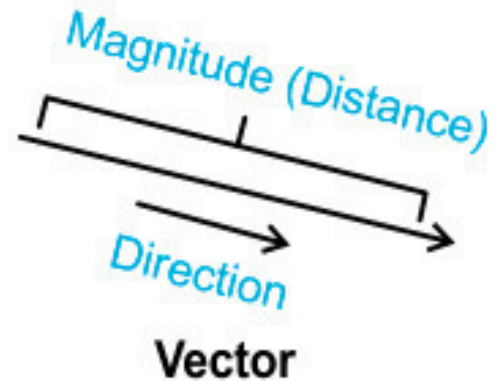
The length of a vector (also called its magnitude) tells us how long or strong the vector is — basically, the size of the vector.

If a vector is written as:

$$\mathbf{A} = \langle x, y \rangle \text{ (in 2D)}$$

then its length is given by:

$$|\mathbf{A}| = \sqrt{x^2 + y^2}$$



Vector Direction (Magnitude of a Vector)

The direction of a vector shows which way it is pointing.

If a vector $\mathbf{A}$ has components x and y , then its direction is the angle it makes with the positive x -axis.

Formula (in 2D):

$$m = \tan^{-1} \frac{y}{x}$$

- Here, θ is the direction angle.
- The result is usually given in degrees or radians.

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3. Vector Equation

The vector equation describes the position of a point or path in vector form, often used in geometry, physics, and coordinate systems.

Position Vector Equation of a Line (2D or 3D)

A line passing through a point A and in the direction of vector d is given by

$$\vec{r} = \vec{a} + \lambda \vec{d}$$

- $\vec{r}$ = position vector of any point on the line
- $\vec{a}$ = position vector of a fixed point on the line
- $\vec{d}$ = direction vector
- λ = scalar parameter (real number)

Problem: Let a point A(1,2) and direction vector = $\langle 3,4 \rangle$. What will be the Vector Equation?

Solution: We know that the equation of vector is:

$$\vec{r} = \vec{a} + \lambda \vec{d}$$

Thus, we can write it as

$$\vec{r} = \langle 1,2 \rangle + \lambda \langle 3,4 \rangle$$

Problem: Let a point A(1,0,2) and direction vector = $\langle 2,-1,3 \rangle$. What will be the Vector Equation?

Solution: We know that the equation of vector is:

$$\vec{r} = \vec{a} + \lambda \vec{d}$$

Thus, we can write it as

$$\vec{r} = \langle 1,0,2 \rangle + \lambda \langle 2,-1,3 \rangle$$

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4. Parallel vectors and Collinear points.

In vector geometry, recognizing relationships between vectors and points is key to solving many problems. Two important concepts are parallel vectors and collinear points, which often go hand in hand.

What Are Parallel Vectors?

Two vectors are called parallel if they move in the same or opposite direction, regardless of their magnitude (length). In simple terms, one is just a stretched or shrunk version of the other.

Mathematical Condition:

Vectors $\vec{a}$ and $\vec{b}$ are parallel if:

$$\vec{a} = k\vec{b}$$

where k is any real number (scalar)

- If $k > 0$: **same direction**
- If $k < 0$: **opposite direction**
- If $k = 0$: **one vector is zero (special case)**

Problem: Are the vectors $P = \langle 4, -2 \rangle$, $Q = \langle 6, -3 \rangle$ parallel?

Solution:

Let's test if there is a scalar k such that:

$$\vec{P} = k\vec{Q}$$

Compare the components:

Let's take the First Component : $4 = k \cdot 6 \Rightarrow k = 4/6 \Rightarrow 2/3$

Now check the second component: $-2 = k \cdot (-3) \Rightarrow k = 2/3 \cdot (-3) \Rightarrow -2$

Both components satisfy the same $k = 2/3$

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What Are Collinear Points?

Points are said to be collinear if they lie along the same straight line. In vector form, this means the vectors formed by those points are parallel.

Vector Rule for Collinearity:

If A, B, and C are three points, they are collinear if:

$$\vec{AB} = k\vec{AC}$$

That is, the vectors from one point to the others are parallel.

Problem: Are the points A(1,2), B(3,6) and C(5,10) collinear?

Solution:

We know that, in order to make three points collinear:

This condition must be satisfied i.e,

$$\vec{AB} = k\vec{AC}$$

Find vectors:

$$\vec{AB} = \langle 3-1, 6-2 \rangle = \langle 2, 4 \rangle$$

$$\vec{AC} = \langle 5-1, 10-2 \rangle = \langle 4, 8 \rangle$$

Now check if they are scalar multiples:

$$\vec{AC} = 2.\vec{AB}$$

Both vectors are parallel. Thus, these three points are Collinear.

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5. Vector Addition and Vector Subtraction.

Vectors can be manipulated just like numbers — but with direction and magnitude in mind. The most common operations are addition, subtraction.

Vector Addition

Vector addition is the process of combining two or more vectors to produce a resultant vector. Vectors have both magnitude and direction, so adding them is not like adding numbers — instead, it involves adding their components.

$$\vec{A} + \vec{B}$$

Geometric Meaning (Tip-to-Tail Method)

- Place the tail of the second vector at the tip of the first vector.
- Draw the resultant vector from the tail of the first vector to the tip of the second vector.
- The result is the vector sum or resultant vector.

This method visually shows how one vector follows another.

Algebraic Method (Component-wise Addition)

If

$$\vec{A} = (a_1, a_2) \text{ and } \vec{B} = (b_1, b_2)$$

Then the sum is:

$$\vec{A} + \vec{B} = (a_1 + b_1, a_2 + b_2)$$

You simply add the corresponding components of the two vectors.

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Important Properties of Vector Addition

- **Commutative Law:**

$$\vec{A} + \vec{B} = \vec{B} + \vec{A}$$

- **Associative Law:**

$$(\vec{A} + \vec{B}) + \vec{C} = \vec{A} + (\vec{B} + \vec{C})$$

- **Identity Element (Zero Vector):**

$$\vec{A} + \vec{0} = \vec{A}$$

- **Inverse Element (Negative Vector):**

$$\vec{A} + (-\vec{A}) = \vec{0}$$

Problem: There are two forces: $\vec{F}_1 = (5, 2)$ N and $\vec{F}_2 = (-1, 4)$ N. Find $\vec{F}_{\text{res}}$?

Solution:

We simply add the vectors component-wi

$$\begin{aligned}\vec{F}_{\text{res}} &= \vec{F}_1 + \vec{F}_2 \\ \vec{F}_{\text{res}} &= (5 + (-1), 2 + 4) \\ \vec{F}_{\text{res}} &= (4, 6) \text{ N}\end{aligned}$$

Therefore, the resultant force comes out to be:

$$\vec{F}_{\text{res}} = (4, 6) \text{ Newtons}$$

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Vector Subtraction

Vector subtraction means finding the difference between two vectors. It helps determine how much one vector differs from another in both direction and magnitude.

Mathematically, subtracting B from A gives a new vector:

$$\vec{A} - \vec{B}$$

Geometric Meaning (Tip-to-Tail Method)

- To visualize $\vec{A} - \vec{B}$, reverse B and add it to $\vec{A}$:
- $\vec{A} - \vec{B} = \vec{A} + (-\vec{B})$
- The resulting vector points from the tip of $\vec{B}$ to the tip of $\vec{A}$.

Algebraic Method (Component-wise Addition)

If

$$\vec{A} = (a_1, a_2) \text{ and } \vec{B} = (b_1, b_2)$$

Then the sum is:

$$\vec{A} + \vec{B} = (a_1 - b_1, a_2 - b_2)$$

You simply add the corresponding components of the two vectors.

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Important Properties of Vector Addition

- **Not Commutative:**

$$\vec{A} - \vec{B} \neq \vec{B} - \vec{A}$$

- **Not Associative:**

$$(\vec{A} - \vec{B}) - \vec{C} \neq \vec{A} - (\vec{B} - \vec{C})$$

- **Identity Element (Zero Vector):**

$$\vec{A} - \vec{0} = \vec{A}$$

- **Inverse Element (Negative Vector):**

$$\vec{A} + (-\vec{A}) = \vec{0}$$

Problem: Let $\vec{A} = (6, 4)$ and $\vec{B} = (2, 5)$. Find $\vec{A} - \vec{B}$ and $\vec{B} - \vec{A}$?

Solution:

We simply subtract the vectors component-wise:

Subtract $\vec{B}$ from $\vec{A}$:

$$\vec{A} - \vec{B} = (6 - 2, 4 - 5) = (4, -1)$$

Subtract $\vec{A}$ from $\vec{B}$:-

$$\vec{B} - \vec{A} = (2 - 6, 5 - 4) = (-4, 1)$$

Therefore, **(4, -1)** and **(-4, 1)** are the resultant answers.

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6. Solved problems related to Vectors.

Example 1: Two vectors are given as

$$\vec{A} = 3\mathbf{i} + 4\mathbf{j} \text{ and } \vec{B} = -2\mathbf{i} + 5\mathbf{j}.$$

Find:

a) $\vec{A} + \vec{B}$

b) The magnitude of the resultant vector.

Solution: We will solve this using the Vector addition method:

Given:

(a): Add $\vec{A}$ and $\vec{B}$ by using vector addition

$$\begin{aligned}\vec{A} + \vec{B} &= (3\mathbf{i} + 4\mathbf{j}) + (-2\mathbf{i} + 5\mathbf{j}) \\ &= (3 - 2)\mathbf{i} + (4 + 5)\mathbf{j} \\ &= 1\mathbf{i} + 9\mathbf{j}\end{aligned}$$

(b): Compute the resultant by using the formula:

$$\begin{aligned}|\vec{R}|^2 &= (a)^2 + (b)^2 \\ |\vec{R}|^2 &= (1)^2 + (9)^2 \\ |\vec{R}|^2 &= 1 + 81 \\ |\vec{R}|^2 &= 82 \\ |\vec{R}| &= (82)^{1/2} \\ |\vec{R}| &= 9.055(\text{approx})\end{aligned}$$

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Example 2 : If

$\vec{A} = 5\mathbf{i} + 3\mathbf{j}$, $\vec{B} = 2\mathbf{i} + 7\mathbf{j}$ find $\vec{A} - \vec{B}$ and its magnitude.

Solution:

Given:

$$\vec{A} = 5\mathbf{i} + 3\mathbf{j}$$

$$\vec{B} = 2\mathbf{i} + 7\mathbf{j}$$

$$\vec{A} - \vec{B} = 5\mathbf{i} + 3\mathbf{j} - (2\mathbf{i} + 7\mathbf{j})$$

$$\vec{A} - \vec{B} = 5\mathbf{i} + 3\mathbf{j} - 2\mathbf{i} - 7\mathbf{j}$$

$$\vec{A} - \vec{B} = 3\mathbf{i} - 5\mathbf{j}$$

$$|\vec{A} - \vec{B}|^2 = (3)^2 + (5)^2$$

$$|\vec{A} - \vec{B}|^2 = 9 + 25$$

$$|\vec{A} - \vec{B}|^2 = 34$$

$$|\vec{A} - \vec{B}| = \sqrt{34}$$

So, the final answer is:

$$\vec{A} - \vec{B} = 3\mathbf{i} - 5\mathbf{j}$$

$$|\vec{A} - \vec{B}| = \sqrt{34}$$

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Example 2 : A line passes through the point A(1,-2,3) and is parallel to the vector $\vec{d} = 2\mathbf{i} + 3\mathbf{j} - \mathbf{k}$. Write the vector equation of the line.

Solution:

Given:

General form of a line:

$$\vec{r} = \vec{a} + \lambda \vec{d}$$

where

$\vec{a}$ = position vector of a point on the line

$\vec{d}$ = direction vector

Here,

$$\vec{a} = 1\mathbf{i} - 2\mathbf{j} + 3\mathbf{k}$$

$$\vec{d} = 2\mathbf{i} + 3\mathbf{j} - \mathbf{k}$$

$$\begin{aligned} \vec{r} &= (1\mathbf{i} - 2\mathbf{j} + 3\mathbf{k}) + \lambda(2\mathbf{i} + 3\mathbf{j} - \mathbf{k}) \\ &= (1 + 2\lambda)\mathbf{i} + (-2 + 3\lambda)\mathbf{j} + (3 - \lambda)\mathbf{k} \end{aligned}$$

So, the final answer is:

$$\vec{r} = (1 + 2\lambda)\mathbf{i} + (-2 + 3\lambda)\mathbf{j} + (3 - \lambda)\mathbf{k}$$