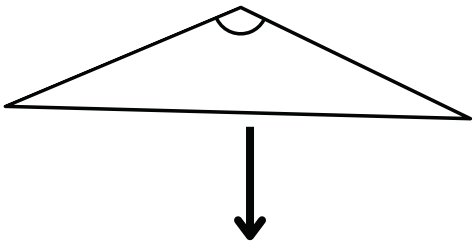
TRIGONOMETRY – GCSE MATHS

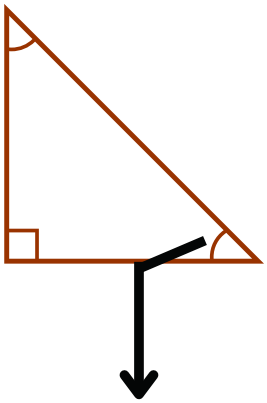
CONTENTS:

1. Introduction

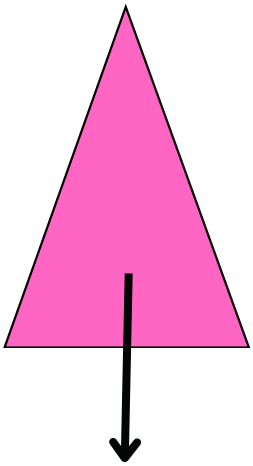
- Trigonometry is all about **Triangles.**
- It is a branch of mathematics that deals with the relationships between the angles and sides of triangles—especially **right-angled triangles**.



Side is missing??



Angle is missing??



Finding the area??

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2. Basics of Trigonometry.

- Trigonometry is the study of the relationship between the **angles** and **sides of triangles**.

Why do we use it?

To find:

- How long a side is
- What an angle is

—when we have the values of some other parts of the triangle.

The Three main Functions:

In a right-angled triangle:

- sin (as: "sine")
- cos (as: "cosine")
- tan (as: "tangent")

They are simply the ratios (fractions) of the given triangle's sides.

All about Triangles

Triangles are three-sided polygons with several important properties. Here are some key properties of triangles:

1. **Basic Properties**

- A triangle has **three sides**, **three vertices**, and **three angles.**
- The sum of the interior angles is always **180°**.
- The sum of the exterior angles is always **360°.**
- Side Length Rule (Triangle Inequality Theorem)

  The sum of any two sides must be greater than the third side:

  a + b > c, b + c > a, a + c > b

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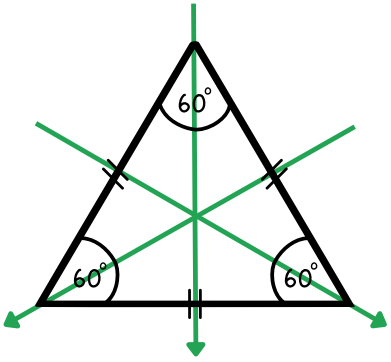
Types of Triangles

There are mainly **four** types of Triangles that can be distinguished uniquely.

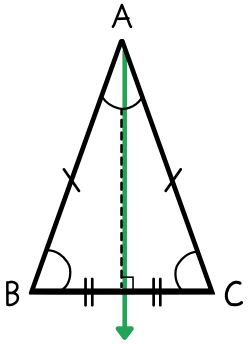
Let us understand about them in detail:

- **Equilateral Triangle**



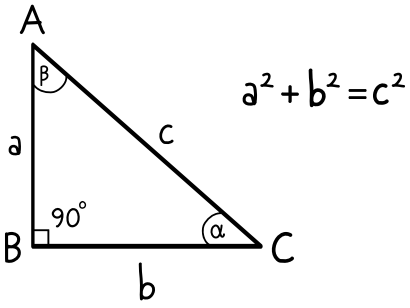
An Equilateral Triangle is a triangle in which all the three sides of the triangle are of the same length.

- **Isosceles Triangle**



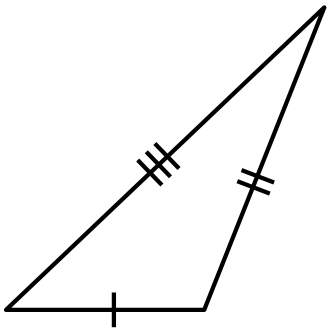
An Isosceles triangle is a triangle that has two sides of equal length. Along with that, the angles opposite those equal sides are also equal.

- **Right Angled Triangle**



A Right-Angled triangle (also called a right triangle) is a triangle that has one angle exactly equal to 90°.

- **Scalene Triangle**



A scalene triangle is a triangle in which all three sides and all three angles are different

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3. Pythagoras' Theorem

$$c^2 = a^2 + b^2$$

- Click here for more detailed information

4. Understanding Sin, Cos, Tan

$$sin(x) = \frac{P}{H} \qquad cos(x) = \frac{B}{H} \qquad tan(x) = \frac{P}{B}$$

- Click here for more detailed information

5. Finding Unknown Sides and Unknown Angles

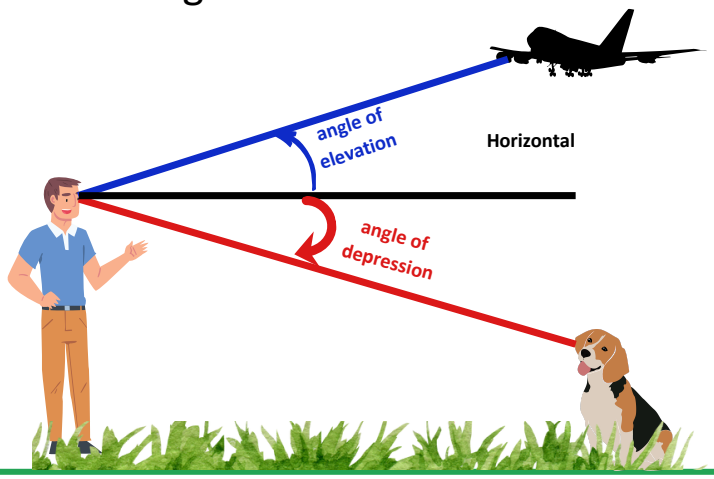
- Click here for more detailed information

6. Angles of Elevation and Depression.

- **Definitions**

Angle of Elevation: The angle formed between the horizontal line (eye level) and the line of sight when an observer looks upwards at an object.

Angle of Depression: The angle formed between the horizontal line (eye level) and the line of sight when an observer looks downwards at an object.



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Angle of Elevation: The angle formed between the horizontal line (eye level) and the line of sight when an observer looks downwards at an object.

- **Key Points**

1. Both angles are measured from the **horizontal** (eye level).
2. They are always between **0° and 90°**.
3. The angle of elevation and depression are **congruent (equal)** when the observer and object are at the same horizontal level (i.e., in symmetric positions).

- **Real Life Applications**

1. Angle of Elevation: Used in measuring heights of buildings, mountains, or trees.
2. Angle of Depression: Used in aviation (pilots landing planes), navigation, or determining distances between objects at different heights.

- **Step by Step Procedure**

Step 1: Draw a Diagram
Step 2: Identify Known & Unknown Values
Step 3: Choose the Right Trigonometric Ratio
Step 4: Solve for the Unknown
Step 5: Check for Angle of Depression

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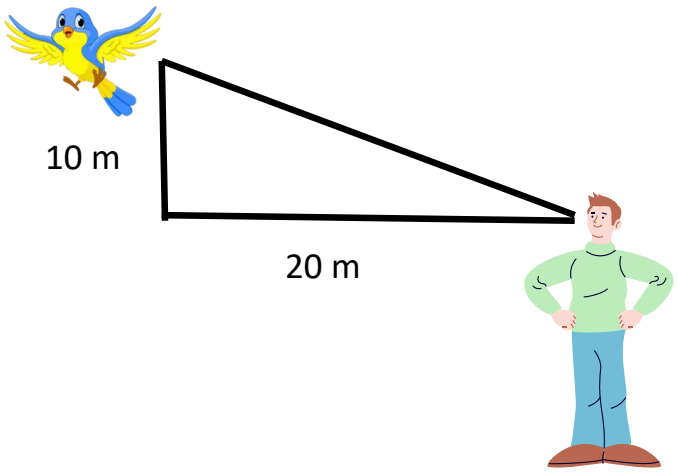
Example:

"A bird sits on a tree 10m high. A man 20 m away looks up at the bird."

Solution:

Step 1: Draw a Diagram

- Sketch the scenario based on the problem statement.
- Label:
  - The observer's eye level (horizontal line).
  - The line of sight (angle of elevation or depression).
  - The height (vertical side) and distance (horizontal side).



Step 2: Identify Known & Unknown Values

- Given:
  - Distance from observer to object (adjacent side).
  - Height (opposite side).
  - Angle (if given).
- Find:
  - The missing side or angle.

Example:

- Given:
  - **Distance (adjacent) = 10m**
  - **Height (opposite) = 15m**
- Find: Angle of elevation (θ).

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Step 3: Choose the Right Trigonometric Ratio

- SOH-CAH-TOA helps decide which ratio to use:
  - Sine (sinθ) = Opposite / Hypotenuse
  - Cosine (cosθ) = Adjacent / Hypotenuse
  - Tangent (tanθ) = Opposite / Adjacent

In our example:

- We have opposite (height) = 10m and adjacent (distance) = 20m.
- Use tangent:
- $\tan\theta = \frac{Opposite}{Adjacent} = \frac{10}{20} =$ **0.5**

Step 4: Solve for the Unknown

- If finding an angle, use inverse trig functions ($\tan^{-1}$, $\sin^{-1}$, $\cos^{-1}$).
- If finding a side, rearrange the formula.

Example (continued):

- To find θ:
- $\vartheta = \tan^{-1}(0.5) \approx$ **26.57°**

Step 5: Check for Angle of Depression

- If the problem involves looking downward, the steps are the same, but the angle is measured below the horizontal.
- **Key Fact:**

  Angle of elevation from point A to B = Angle of depression from B to A (they are equal due to alternate angles).

Therefore,

Angle of Elevation = Angle of Depression

Hence,

Angle of depression ≈ **26.57°**

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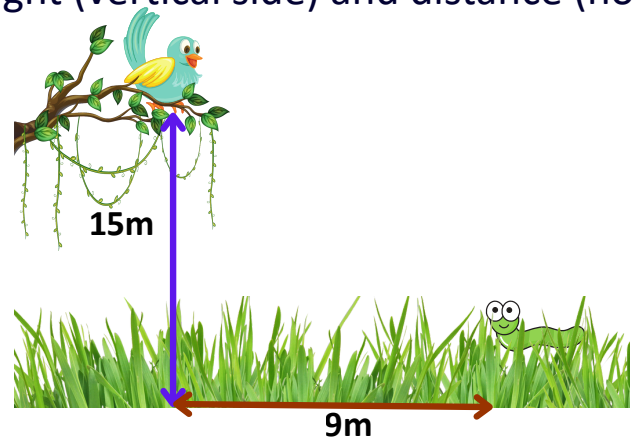
Example:

A bird is perched on a 15-meter-high tree. It spots a worm on the ground 9 meters away from the base of the tree. What is the angle of depression from the bird to the worm?

Solution:

Step 1: Draw a Diagram

- Sketch the scenario based on the problem statement.
- Label:
  - The observer's eye level (horizontal line).
  - The line of sight (angle of elevation or depression).
  - The height (vertical side) and distance (horizontal side).



Step 2: Identify Known & Unknown Values

- Given:
  - Distance from observer to object (adjacent side).
  - Height (opposite side).
  - Angle (if given).
- Find:
  - The missing side or angle.

Example:

- Given:
  - Distance (adjacent) = 9m
  - Height (opposite) = 15m
- Find: Angle of depression(θ).

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Step 3: Choose the Right Trigonometric Ratio

- SOH-CAH-TOA helps decide which ratio to use:
  - Sine (sinθ) = Opposite / Hypotenuse
  - Cosine (cosθ) = Adjacent / Hypotenuse
  - Tangent (tanθ) = Opposite / Adjacent

In our example:

- We have opposite (height) = 15m and adjacent (distance) = 9m.
- Use tangent:
- $\tan\theta = \frac{Opposite}{Adjacent} = \frac{15}{9} =$ **1.67**

Step 4: Solve for the Unknown

- If finding an angle, use inverse trig functions ($\tan^{-1}$, $\sin^{-1}$, $\cos^{-1}$).
- If finding a side, rearrange the formula.

Example (continued):

- To find θ:
- $\vartheta = \tan^{-1}(1.67) \approx$ **59.3°**

Step 5: Check for Angle of Elevation

- If the problem involves looking downward, the steps are the same, but the angle is measured below the horizontal.
- Key Fact:

Angle of elevation from point A to B = Angle of depression from B to A (they are equal due to alternate angles).

Therefore,

Angle of Elevation = Angle of Depression

Hence,

Angle of depression ≈ 59.3°

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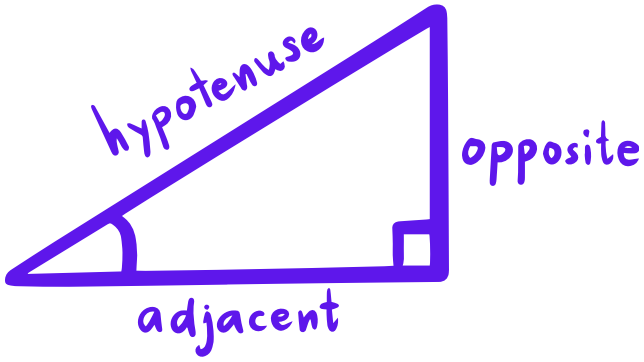
8. Triangle Exact Values



Let us understand about some important ratios in brief:

- **Sine (sinθ) = Opposite / Hypotenuse**
- **Cosine (cosθ) = Adjacent / Hypotenuse**
- **Tangent (tanθ) = Opposite / Adjacent**

where the terms are denoted as:

1. **Opposite** = side opposite the angle
2. **Adjacent** = side next to the angle (not the hypotenuse)
3. **Hypotenuse** = the longest side (opposite the 90° angle)

θ (degrees)	sin(θ)	cos(θ)	tan(θ)
0°	0	1	0
30°	1/2	√3/2	1/√3 ≈ 0.577
45°	√2/2	√2/2	1
60°	√3/2	1/2	√3 ≈ 1.732
90°	1	0	undefined

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Tip: We have to summarize this table given above to solve each of the question accurately

Examples

Problem: In a right triangle, the angle is 30° and the adjacent side is 6 units. Find the opposite side.

Solution:

$$\tan 30° = \frac{Opposite}{Adjacent}$$

$$\frac{1}{\sqrt{3}} = \frac{x}{6}$$

$$x = \frac{6}{\sqrt{3}}$$

$$x = \sqrt[2]{3}$$

So, therefore we got an answer to our question that is:

$$\boxed{x = \sqrt[2]{3}}$$

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Problem: In a right triangle, the angle is 30° and the opposite side is 9 units. Find the opposite side.

Solution:

Given:

Angle = 30°

Adjacent side = 6 units

We know that

$$\tan 30° = \frac{Opposite}{Adjacent}$$

$$\frac{1}{\sqrt{3}} = \frac{9}{x}$$

$$x = \sqrt[9]{3}$$

So, therefore we got an answer to our question that is:

$$x = \sqrt[9]{3}$$

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9. Three Solved Problems

Example 1: A shed roof makes an angle of 41° with the horizontal. Given that the width of the shed is 6 m and the length of its slope is 4 m. Calculate the height of the roof.

Solution:

Given:

- **Angle (θ)** = 41° (between the roof and the horizontal)
- **Slope length (L)** = 4 m (the hypotenuse of the right triangle formed by the roof)
- **Width (W) =** 6 m (total horizontal span of the shed)

The width of the shed (6 m) is the total span, but the roof slope only covers half of this (since it's a symmetrical shed roof).

$$x = W/2$$
$$= 6/2$$
$$= 3 \text{ m}$$

$$\sin 41° = \frac{Opposite}{Hypotenuse}$$

$$0.6561 = \frac{h}{4}$$

$$h = 0.6561 \times 4$$
$$h = 2.624 \text{ m}$$

The height of the roof is approximately 2.624 meters.

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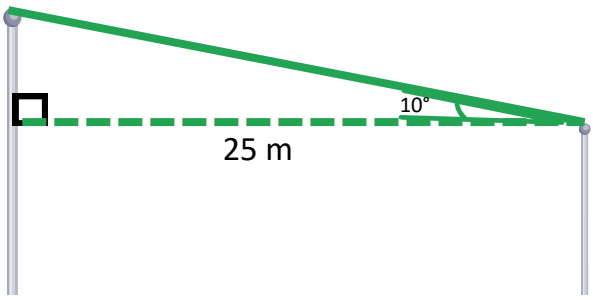
Example 2: A zip wire runs between two poles 45m apart. The zip wire is at an angle of 10° to the horizontal. Calculate the length of the zip wire.



Solution:

Given:

- **Angle (θ)** = 10° (between the zip wire and the length)
- **Width (W) =** 25 m (Distance between two poles)

The width of the shed (6 m) is the total span, but the roof slope only covers half of this (since it's a symmetrical shed roof).

$$\cos 10° = \frac{Adjacent}{Hypotenuse}$$

$$0.9848 = \frac{25}{l}$$

$$L = \frac{25}{0.9848}$$

$$L = 25.38 \text{ m}$$

The length of the zip wire is approximately 25.38 meters.

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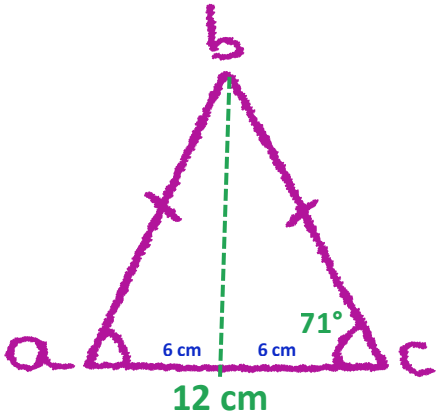
Example 3: Triangle ABC is an isosceles. Calculate the height of the given triangle.



Solution:

Given:

- **Angle (θ)** = 71° (between the two sides)
- **Side length =** 12 cm (Distance between two poles)

The width of the shed (6 m) is the total span, but the roof slope only covers half of this (since it's a symmetrical shed roof).

$$x = W/2$$
$$= 6/2$$
$$= 3 \text{ m}$$

$$\tan 71° = \frac{Opposite}{Adjacent}$$

$$2.9042 = \frac{h}{6}$$

$$h = 17.4 \text{ cm}$$

The height of the triangle ABC is approximately 17.4 centimeters.