Simultaneous Equations - GCSE Maths

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1. Introduction

- Two or more equations are solved together at the same time.

- A solution of these equation is the value of variables which satisfy both of the equations.

  Examples:

  1) $5x - 9y = 1$

  2) $2x + 5 = 9$

  3) $x + y = 10$

2. What are types of Simultaneous Equations?

- Simultaneous equations can be **linear** (where variables are raised to the power 1) or **non linear** (exponent more than 1).

- Solution of Simultaneous equation: values of variables that satisfy both the equations

- Simultaneous equations are used in various fields like mathematics, physics, computer science and economics.

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Simultaneous Equations

Linear

$ax + b = 0$

$10x + 9 = 0$

Non-Linear

$ax^2 + bx + c = 0$

$5x^2 - 9x + 33 = 0$
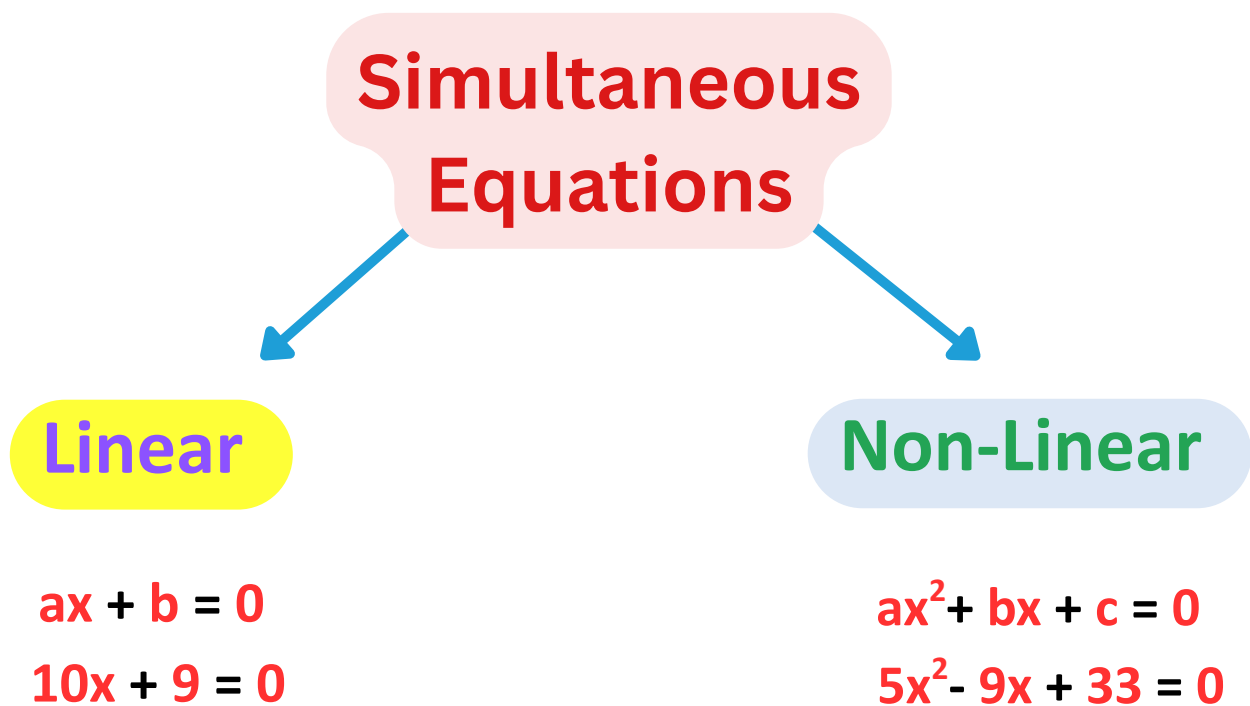
3. What are the methods to solve Simultaneous Equations?

There are two methods to solve Simultaneous Equations-

- **Elimination method**: In this method, we **eliminate one of the variables** by adding or subtracting the equations, which results in a simpler equation with only one variable.

- **Substitution method** : In this method one **variable is expressed in terms of the other variable** and then its value is substituted in other equation, which results in a simpler equation with one variable.

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4. Steps to Solve Simultaneous equations by Elimination method

Elimination method

Let us solve the following simultaneous equations step by step using the elimination method:

Step#1: Identify the variable that can be eliminated :

We can see that if we multiply the 2nd equation with 2 then the coefficient of x in both the equations will become equal and then it can be eliminated by subtraction :

$$2x + 3y = 5 \quad ...(1)$$
$$2x - y = 0 \quad ...(2)$$

Step#2: Add or subtract the equations to get a combined equation with only one variable:

Here, one variable (x) will get eliminated if we subtract equation 2 from 1:

$$2x + 3y = 5 \quad ...(1)$$
$$- [2x - 2y = 0] \quad ...(2)$$

(2x - 2x = 0)	[3x -(-2y) = 5y]	(5 - 0 = 5)

The combined equation is :

$$5y = 5$$
$$y = 1$$

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Step#3: Use the known variable's value to find the other variable's value :

Pick one of the simultaneous equations and put the value of y in it -

$$2x + 3y = 5$$
$$2x + 3(1) = 5$$
$$2x = 5 - 3$$
$$2x = 2$$
$$x = 1$$

So, the solution of these simultaneous equations is:

$$x = 1 \text{ and } y = 1$$

5. Steps to solve Simultaneous equations by Substitution method -

Substitution method :

Let us solve the pair of Simultaneous equations by using Substitution method :

$$4y - x = 9$$
$$3y + 11 = 20$$

Step#1: Find one variable's value in the form of the other variable:

Example:

$$4y - x = 9 \qquad ...(1)$$
$$3y + 11 = 20 \qquad ...(2)$$

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From 1st Equation : $4y - x = 9$

$4y = 9 + x$

$y = \dfrac{(9 + x)}{4}$

Step#2: Put the value in the 2nd equation:

$3y + 11 = 20$

$\dfrac{3(x+9)}{4} + \dfrac{11}{4} = 20$

$3x + 27 + 44 = 20 \times 4$ **(taking LCM)**

$3x + 71 = 80$

$3x = 80 - 71$

$x = \dfrac{9}{3}$

$x = 3$

Step#3: Use the known value of one variable to find the value of

the other-

Pick one of the original Equations:

From 1st equation: $4y - x = 9$

put x = 3 $4y - 3 = 9$

$y = \dfrac{12}{4}$

$y = 3$

thus, the solution for these simultaneous equation is

$y = 3$ and $x = 3$

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5. Solved Examples

Problem#1:

Solve the following pair of Simultaneous equations by using the Elimination method

$$3x + 2y = 12$$
$$x - 2y = 4$$

Step#1: Identify the variable that can be eliminated :

$$3x + 2y = 12 \quad\quad ...(1)$$
$$x - 2y = 4 \quad\quad ...(2)$$

Multiply equation 2nd by 3 so that the coefficients of x in both the equations become equal.

$$3x + 2y = 12 \quad\quad ...(1)$$
$$3 \times (x - 2y = 4) \quad\quad ...(2)$$

The resulting equations are :

$$3x + 2y = 12 \quad\quad ...(1)$$
$$3x - 6y = 12 \quad\quad ...(2)$$

Step#2: Subtract the 2nd from the 1st:

$$3x + 2y = 12 \quad\quad ...(1)$$
$$- [3x - 6y = 12] \quad\quad ...(2)$$

(3x - 3x = 0)	[2y -(- 6y) = 8y]	(12 - 12 = 0)

The combined equation: **8y = 0** thus, **y = 0**

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Step#3: Use the known value of one variable to find the value of the other variable -

Pick one of the original Equations-

$$3x + 2y = 12$$

Put y = 0

$$3x + 2(0) = 12$$

$$3x + 0 = 12$$

$$x = \frac{12}{3}$$

$$x = 4$$

Thus, the solution is : **x = 4** and **y = 0**

Problem#2

Solve the pair of Simultaneous equations by Substitution method.

$$2x - y = 3$$
$$x + y = 9$$

Step#1: Find one variable's value in the form of the other variable:

From 1st Equation-

$$2x - y = 3 \quad ...(1)$$

$$2x = 3 + y$$

$$x = \frac{(3 + y)}{2}$$

Step#2: Put the value in the 2nd equation:

$$x + y = 9 \quad ...(2)$$

$$\frac{(3 + y)}{2} + y = 9$$

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$$\frac{(3 + y)}{2} + \frac{2y}{2} = 9$$

$$3 + y + 2y = 9 \times 2$$

$$3 + 3y = 18$$

$$3y = 18 - 3$$

$$y = \frac{15}{3}$$

$$y = 5$$

Step#3: Use the known variable's value to find the other variable:

From 1st equation $\qquad 2x = 3 + y$

$$2x = 3 + 5$$

$$x = \frac{8}{2}$$

$$x = 4$$

Thus, the solution is: **x = 4** and **y = 5**

Problem#3

Solve the pair of simultaneous equations by elimination method:

$$4x + y = 3$$

$$3x - 5y = 8$$

Step#1: Identify the variable that can be eliminated : Here, if we multiply the 1st equation with 3 and 2nd equation with 4 then the coefficient of x in both the equations will become equal -

$$3(4x + y = 3) \qquad \ldots(1)$$

$$4(3x - 5y = 8) \qquad \ldots(2)$$

and the 1st equation becomes -

$$12x + 3y = 9 \qquad \ldots(1)$$

$$12x - 20y = 32 \qquad \ldots(2)$$

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Step#2: Subtract the 2ⁿᵈ equation from 1ˢᵗ -

$$12x + 3y = 9 \qquad ...(1)$$

$$-[12x - 20y = 32] \qquad ...(2)$$

$(12x - 12x = 0)$ $[3y - (-20y) = 23y)]$ $(9 - 32 = -23)$

The combined equation is :

$$23y = -23$$

$$y = -1$$

Step#3: Use the known variable's value to find other -

From equation (1): $4x + y = 3$

put y = 1 $\qquad 4x - 1 = 3$

$$4x = 4$$

$$x = 1$$

Hence, the solutions of these simultaneous equations are :

$$x = 1 , y = -1$$

Let us check whether the values satisfy the 2ⁿᵈ equation or not :

From 2ⁿᵈ equation -

$$3x - 5y = 8$$

(put x = 1, y = -1) $\quad 3(1) - 5(-1) = 8$

$$3 + 5 = 8$$

$$8 = 8$$

(Hence, the solution is correct)