Quadratic Simultaneous Equations

Contents

1. Introduction

- Two or more equations that share variables and have set of values for variables that satisfy both the equations.
- If the maximum power raised to the variable in Simultaneous equations is one then these are called Linear Simultaneous Equations. Similarly If the power raised to the variables is two they are called Quadratic Simultaneous Equations.

Quadratic Simultaneous Equations

2. Step by Step Solving Quadratic Simultaneous Equations

Example :

$$y = 2x - 1$$
$$y = x^2 + 3x + 5$$

Step#1: Substitute the value of one variable from linear equation(to eliminate one variable) -

Putting y = 2x -1 in 2nd equation -

$$2x - 1 = x^2 + 3x + 5$$
$$x^2 + 3x + 5 - 2x + 1 = 0$$
$$x^2 + 1x + 6 = 0$$
$$x^2 + 3x - 2x + 6 = 0$$
$$x(x + 3) - 2(x + 3) = 0$$
$$(x + 3)(x - 2) = 0$$
$$x = -3, x = 2$$

Step#2: Substitute the values obtained back into the original equation and find possible values of another variable-

For x = -3 from 1st Equation-

$$y = 2x - 1$$
$$y = 2 \times (-3) - 1$$
$$y = -6 - 1$$
$$y = -7$$

For x = 2 from 1st Equation -

$$y = 2x - 1$$
$$y = 2 \times (2) - 1$$
$$y = 4 - 1$$
$$y = 3$$

Step#3: Final Answer - x = -3 and y = -7

OR x = 2 and y = 3

Quadratic Simultaneous Equations

3. Solved Examples

Example(1)

$$2x - y = 7$$
$$x^2 - 7x + y = 7$$

Solution:

Step#1: From 1st Equation, $2x - y = 7$

$y = 2x - 7$

Put in 2nd Equation, $x^2 - 7x + (2x - 7) = 7$

$x^2 - 7x + 2x - 7 - 7 = 0$

$x^2 - 5x - 14 = 0$

$x^2 - 7x + 2x - 14 = 0$

$x(x - 7) + 2(x - 7) = 0$

$(x + 2)(x - 7) = 0$

$x = -2, x = 7$

Step#2: Put x = -2 in 1st Equation -

$2 X (-2) - Y = 7$

$-4 - y = 7$

$y = -4 - 7$

$y = -11$

Put x = 7 in 1st Equation -

$2 X (7) - y = 7$

$14 - y = 7$

$y = 7$

Step#3: Final Answer is - x = -2 and y = -11

OR x = 7 and y = 7

Quadratic Simultaneous Equations

Example (2)

$$x + y = 12$$
$$x^2 + y^2 = 80$$

Solution:

Step#1: From 1st Equation - $x + y = 12$

$$x = 12 - y$$

Substitute in 2nd Equation -

$$x^2 + y^2 = 80$$
$$(y - 12)^2 + y^2 = 80$$
$$y^2 + 144 - 24y + y^2 = 80$$
$$2y^2 - 24y + 144 - 80 = 0$$
$$2y^2 - 24y + 64 = 0$$

Divide by 2 on both the sides-

$$y^2 - 12y + 32 = 0$$

Factorisation- $y^2 - 8y - 4y + 32 = 0$

$$y \times (y - 8) - 4 \times (y - 8) = 0$$
$$(y - 4)(y - 8) = 0$$
$$y = 4, y = 8$$

Step#2: Put the values of y in original equation to find possible values of x(Using 1st Equation)-

At y = 4, $x + y = 12$

$$x + 4 = 12$$
$$x = 8$$

At y = 8, $x + y = 12$

$$x + 8 = 12$$
$$x = 4$$

Step#3: Final Answer is x = 8 and y = 4

OR x = 4 and y = 8

Quadratic Simultaneous Equations

Example(3): Solve the following example-

$$8x + y = 42$$
$$x^2 - y = 6$$

Solution:

Step#1: From 1st equation - $8x + y = 42$

$$y = 42 - 8x$$

Substitute it in 2nd Equation-

$$x^2 - y = 6$$
$$x^2 - 42 - 8x = 6$$
$$x^2 - 8x - 36 = 0$$
$$x^2 - 12x + 3x - 36 = 0$$
$$x \times (x - 12) + 3 \times (x - 12) = 0$$
$$(x + 3)(x - 12) = 0$$
$$x = -3, 12$$

Step#2: Find the possible values of y (from 1st equation)-

At x = -3,

$$8x + y = 42$$
$$8 \times (-3) + y = 42$$
$$-24 + y = 42$$
$$y = 18$$

At x = 12,

$$8x + y = 42$$
$$8 \times (12) + y = 42$$
$$96 + y = 42$$
$$y = -54$$

Step#3: Final answer is x = -3 and y = 18 OR

x = 12 and y = -54

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Example(4)

$$2x^2 + y = 5$$
$$2x + y = 1$$

Solution:

Step#1: From 2nd Equation- $2x + y = 1$

$$y = 1 - 2x$$

Substitute in 1st Equation -

$$2x^2 + y = 5$$
$$2x^2 + 1 - 2x = 5$$
$$2x^2 - 2x - 4 = 0$$

Divide by 2 on both sides -

$$x^2 + x - 2 = 0$$
$$x^2 + 2x - x - 2 = 0$$
$$x \times (x + 2) - 1 \times (x + 2) = 0$$
$$(x - 1)(x + 2) = 0$$
$$x = 1, -2$$

Step#2: Use the values of x to find possible values of y from 2nd equation, At x = 1, $2 \times (1) + y = 1$

$$y = -1$$

At x = -2 $2 \times (-2) + y = 1$

$$y = 5$$

Step#3: Final answer is x = 1 and y = -1 OR

x = -2 and y = 5

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Example(5)

$$y + 3x = 8$$
$$y = 2x + x^2 + 4$$

Solution:

Step#1: From 1st Equation- $y + 3x = 8$

$$y = 8 - 3x$$

Step#2: Substitute in 2nd Equation -

$$y = x^2 + 2x + 4$$
$$8 - 3x = x^2 + 2x + 4$$
$$x^2 + 2x + 4 - 8 + 3x = 0$$
$$x^2 + 5x - 4 = 0$$
$$x^2 + 4x - x - 4 = 0$$
$$x(x + 4) - 1(x + 4) = 0$$
$$(x - 1)(x + 4) = 0$$
$$x = 1, -4$$

Step#2: By using the values of x we can find possible values of y -

From 1st Equation,

At x = 1, $y + 3x = 8$

$$y + 3 \times (1) = 8$$
$$y = 8 - 3$$
$$y = 5$$

At x = -4, $y + 3x = 8$

$$y + 3 \times (-4) = 8$$
$$y = 8 + 12$$
$$y = 20$$

Step#3: Final answer is x = 1 and y = 5

OR x = -4 and y = 20