

OPERATIONS WITH FRACTIONS – GCSE

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1. Introduction

Operations are the basic processes used to manipulate numbers and expressions. The four fundamental operations are:

- **Addition (+)**
- **Subtraction (-)**
- **Multiplication (x)**
- **Division (÷)**

2. Operations with Fractions

In mathematics, an **Operation** is a process or action that **produces a new value from one or more inputs**, such as addition, subtraction, multiplication, or division.

Order of Operations:

To solve expressions correctly, follow the order:

Parentheses → Exponents → Multiplication/Division → Addition/Subtraction

OPERATIONS WITH FRACTIONS – GCSE MATHS

1. Addition of Fractions

1. Same Denominator

If the denominators (bottom numbers) are the same, just add the numerators (top numbers):

Example:

$$\frac{3}{4} + \frac{2}{4} = \frac{6}{4}$$

2. Different Denominators

If the denominators are different, follow these steps:

Step #1: Find the Least Common Denominator (LCD)

The smallest number that both denominators can divide into.

Step #2: Convert fractions to have the same denominator

Step #3: Add the numerators

Step #4: Simplify the result (if needed)

Example:

$$\begin{aligned}\frac{3}{4} + \frac{7}{8} &= \frac{3(8) + 7(4)}{32} = \frac{52}{32} \\ &= \frac{13}{8}\end{aligned}$$

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Problem: Convert $\frac{3}{4} + \frac{7}{2}$ as a fraction

Solution:

Step #1: Make the bottom numbers the same

$$\begin{aligned} &= \frac{3}{4} + \frac{7}{2} \\ &= \frac{3 + 14}{4} \end{aligned}$$

Step #2: Add the top numbers

$$= \frac{17}{4}$$

Step #3: Convert back to a Mixed Fraction

$$= 3\frac{5}{4}$$

2. Subtraction of Fractions

1. Same Denominator

If the denominators (bottom numbers) are the same, subtract the numerators (top numbers):

Example:

$$\frac{9}{4} - \frac{2}{4} = \frac{7}{4}$$

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2. Different Denominators

If the denominators are different, follow these steps:

Step #1: Find the Least Common Denominator (LCD)

The smallest number that both denominators divide into.

Step #2: Convert fractions to have the same denominator

Step #3: Subtract the numerators

Step #4: Simplify the result (if possible)

Example:

$$\begin{aligned}\frac{3}{8} - \frac{1}{3} &= \frac{3(3) - 8(1)}{24} \\ &= \frac{1}{24}\end{aligned}$$

Problem: Convert $9/4 - 5/2$ as a fraction

Solution:

Step #1: Make the bottom numbers the same

$$= \frac{9}{4} - \frac{5}{3} = \frac{9(3) - 5(4)}{12}$$

Step #2: Subtract the top numbers

$$= \frac{7}{12}$$

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3. Multiplication of Fractions

Multiplying fractions is straightforward—just multiply numerators together and denominators together, then simplify if possible.

1. Basic Rule: For a problem ,such as $\frac{a}{b} \times \frac{c}{d} = \frac{a \times c}{b \times d}$

- Numerator of the product = $a \times c$
- Denominator of the product = $b \times d$

2. Steps with Simplification

Step #1: Multiply the numerators: $a \times c$.

Step #2: Multiply the denominators: $b \times d$.

Step #3: Simplify the resulting fraction by dividing numerator and denominator by their **greatest common divisor (GCD)**.

Example: Multiply $\frac{2}{3} \times \frac{6}{5}$

$$\frac{2}{3} \times \frac{6}{5} = \frac{2 \times 6}{3 \times 5}$$

$$= \frac{12}{15}$$

$$= \frac{4}{5}$$

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Problem: Convert $8/3 \times 6/5$ as a fraction

Solution:

Step #1: Multiply top numbers together.

$$= \frac{8}{3} \times \frac{6}{5}$$

Step #2: Multiply bottom numbers together.

$$= \frac{8 \times 6}{3 \times 5}$$

Step #3: Simplify the result

$$= \frac{48}{15}$$

4. Division of Fractions

Dividing fractions involves multiplying by the reciprocal (or “flip”) of the divisor.

1. Basic Rule: For a problem ,such as $\frac{a}{b} \div \frac{c}{d} = \frac{a \times d}{b \times c}$

- Numerator of the product = $a \times d$
- Denominator of the product = $b \times c$

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2. Steps with Simplification

Step #1: Write the problem: $\frac{a}{b} \div \frac{c}{d}$

Step #2: Reciprocal, Change to $\frac{a}{b} \div \frac{c}{d} = \frac{a \times d}{b \times c}$

Step #3: Multiply numerators and denominators. Simplify the result by dividing numerator and denominator by their greatest common divisor (GCD).

Example: Divide $\frac{3}{4} \div \frac{2}{7}$

$$\frac{3}{4} \times \frac{7}{2} = \frac{21}{8}$$

$$= \frac{21}{8}$$

Problem: Convert $3/4 \div 7/2$ as a fraction

Solution:

Step #1: Keep the first fraction same and change the divide sign to multiplication sign and reciprocate the second fraction.

$$= \frac{3}{4} \times \frac{2}{7}$$

Step #2: Multiply bottom and top numbers together.

$$= \frac{6}{28}$$

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6. Solved Examples

Problem: Emma baked $\frac{2}{3}$ of a tray of cookies in the morning and $\frac{1}{4}$ of a tray in the afternoon. How much of a full tray did she bake in total?

Solution:

Step #1: Write down the given information

In Morning, Emma baked:- $\frac{2}{3}$

At afternoon, fraction of tray gets completed:- $\frac{1}{4}$

Step #2: Simplify to make a common denominator

We know that:

$$\begin{aligned}\frac{2 \times 4}{3 \times 4} &= \frac{8}{12} \\ \frac{1 \times 3}{4 \times 3} &= \frac{3}{12}\end{aligned}$$

Step #3: Calculate the final result by applying favorable operations

The total amount of baking that has been completed: $= \left(\frac{8}{12} + \frac{3}{12} \right)$

$$= \frac{8 + 3}{12} = \frac{11}{12}$$

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Problem: A ribbon is $2\frac{3}{4}$ meter long. You need pieces of length $\frac{1}{6}$ meter. How many full pieces can you cut?

Solution:

Step #1: Write down the given information

$$\text{Length of ribbon: } 2\frac{3}{4} = \frac{11}{4}$$

$$\text{Pieces of length: } \frac{1}{6}$$

Step #2: Divide total length by piece length

We know that:

$$\begin{aligned} &= \frac{\frac{11}{4}}{\frac{1}{6}} \\ &= \frac{11 \times 6}{4 \times 1} \end{aligned}$$

Step #3: Calculate the final result by applying favorable operations

$$\text{Pieces that we can cut from } 11/4\text{m length of ribbon: } \frac{11 \times 6}{4 \times 1} = \frac{66}{4}$$

$$= \frac{66}{4} = \frac{33}{2} = 16.5$$

Therefore, we can cut 16 full pieces from that ribbon.