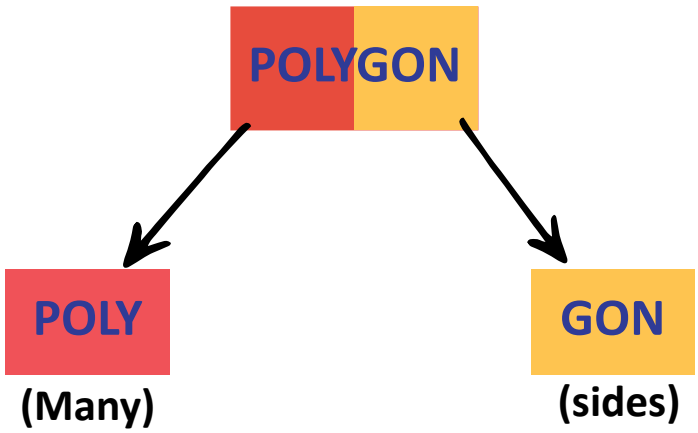
Interior and Exterior Angles in Polygons

CONTENTS

1. Introduction

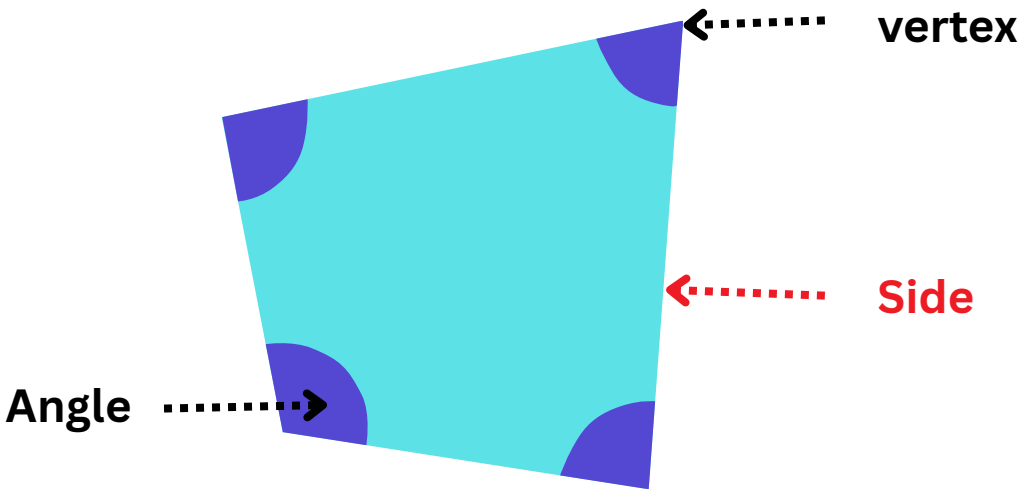
Word **Polygon** is made up of two words -



- A **closed shape** made of line segments .
- To make a Polygon **minimum three line segments** are required which end up making a Triangle.
- Basic Polygons are **Triangle, Square** and **Rectangle**. Polygons have vertices, angles and sides.
- Angle is basically distance between two rays starting at the same point.
- Polygons have two types of angles, they are - **Interior and Exterior angles.**

Interior and Exterior Angle in Polygon

- Polygons are 2-Dimensional shapes and we can use them to make 3-Dimensional objects.
- Polygons have vertices, side and angles.



Importance of polygons:

- Polygons play vital role in understanding the **geometric concepts** like shape, angles, area and perimeter.
- Polygons are present in our **daily life** buildings, houses and designs of objects.
- Students get to know about **angle measurement** and length measurement like concepts which are applicable to solve real-world problems and makes math meaningful.

2. Types of Polygons

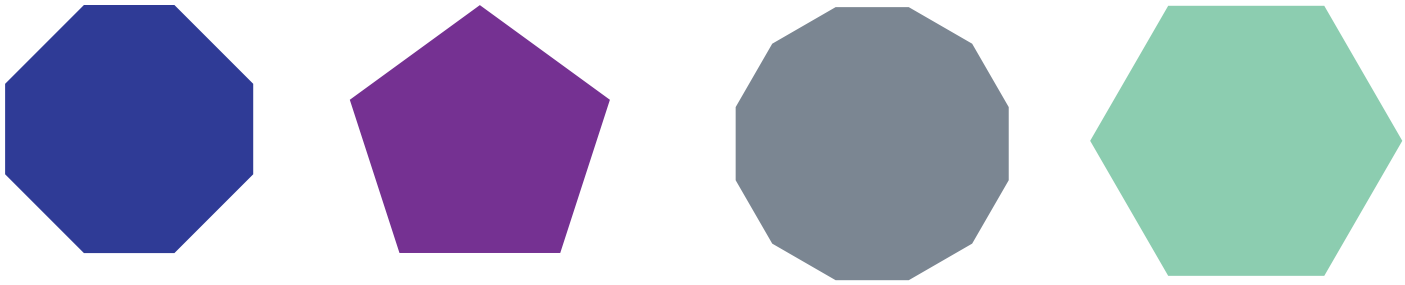
Polygons are classifies into two types -

1. Regular Polygons
2. Irregular Polygons

Interior and Exterior Angles in Polygons

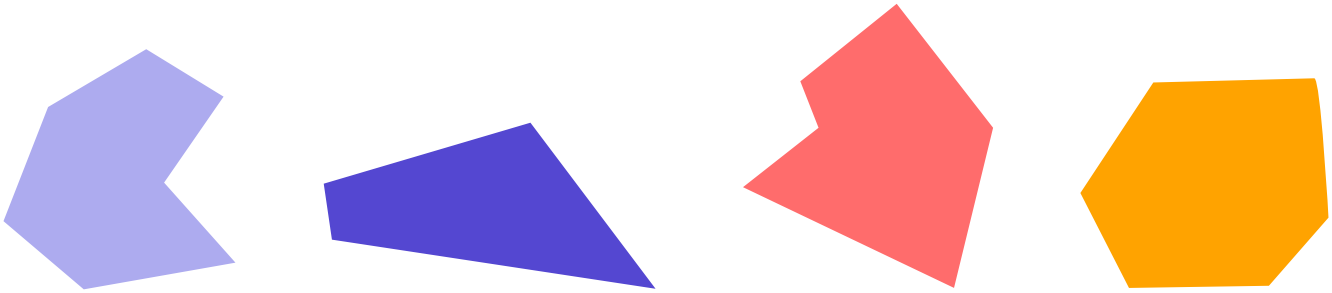
Regular Polygons

The Polygons with equal side and equal Angles



Irregular Polygons

The Polygons with unequal side and unequal Angles



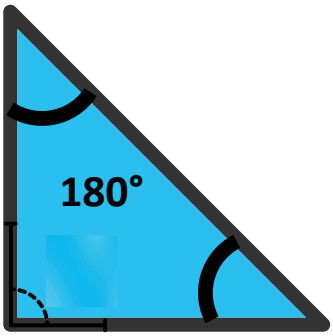
Some important Polygons are as follows -

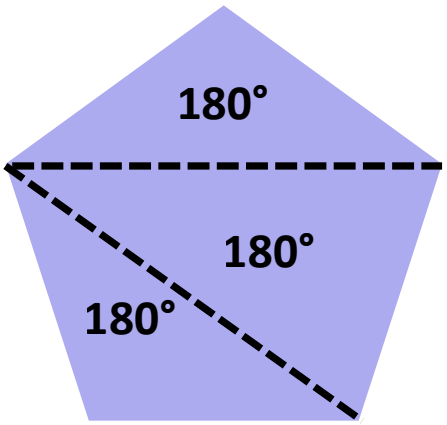
Quadrilateral	Pentagon	Hexagon	Heptagon

Interior and Exterior Angles in Polygons

3. Interior Angles in Polygons

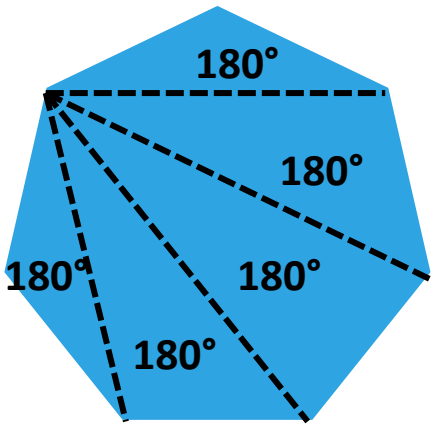
- The **Angles present inside the polygon** are known as Interior Angles.
- The polygon with **minimum number of sides** is **Triangle** and the sum of interior angles of Triangle is 180 degree. Consider other Polygons divided into triangles -





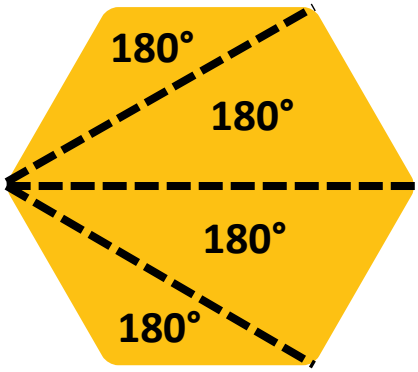
(5 - 2) Triangles

Pentagon



(7 - 2) Triangles

Heptagon



(6 - 2) Triangle

Hexagon

Interior and Exterior Angles in Polygons

- We can conclude that every Polygon can be divided into triangles, which are two fewer than the number of sides of the Polygon, in such a way that the angles are in the form -

$$(n - 2) \times 180°$$

Where n = Number of sides of the Polygon

Example: The Pentagon have 5 sides so -

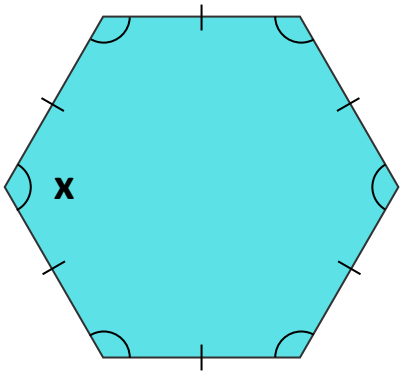
$$(5 - 2) \times 180 = 540°$$

- If we want to find angle of a regular polygon -

$$\frac{(n - 2) \times 180°}{n}$$

Solved examples

Example: Find the missing interior angles in the following Polygons



Solution: The polygon shown in the diagram is an Hexagon. In which -

$$(6 - 2) \times 180° = 720°$$

As the Polygon is a regular hexagon, thus every angle is equal to -

$$x = \frac{720°}{6}$$

$$x = 120°$$

Interior and Exterior Angles in Polygons

Example: Work out the size of angle for the following value of n (Number of sides of Regular Polygon)

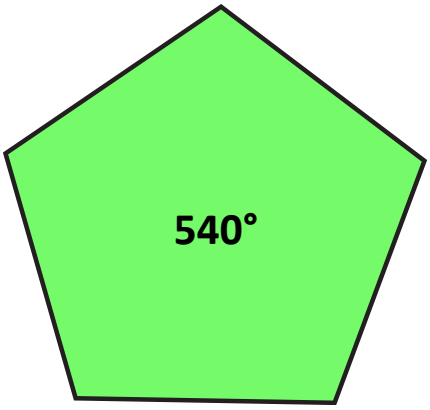
n = 5

Solution: Using formula- **(n-2) X 180°**

Step#1: Find the sum of all interior angles -

Put (n = 5)

A Polygon with 5 sides is called a Pentagon.



(5-2) X 180° = 3 X 180°

540°

Step#2: Find the value of a single angle of a polygon -

Divide the sum of angles by number of sides :

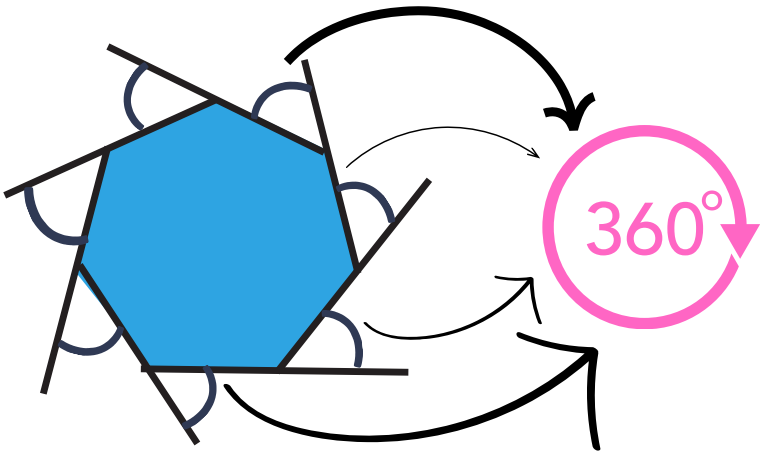
$$x = \frac{540°}{5}$$

x = 108°

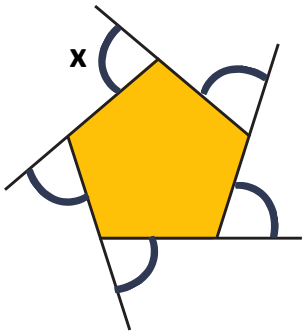
Interior and Exterior Angles - GCSE Maths

4. Exterior Angle in Polygons

- When we **extend** any side of a Polygon, then the angle made is called as **exterior Angle**.
- When the exterior angles are combined together they **form a circle** which represents a complete angle of **360 degrees**. The angles shown below are Exterior angles.



- In the the following diagram a Regular Pentagon, we know exterior angles summed up together gives us 360 degrees.



- Relationship between exterior angle and sides of polygon -

$$\text{Exterior angle} = \frac{360°}{n}$$

$$x + x + x + x + x = 360°$$

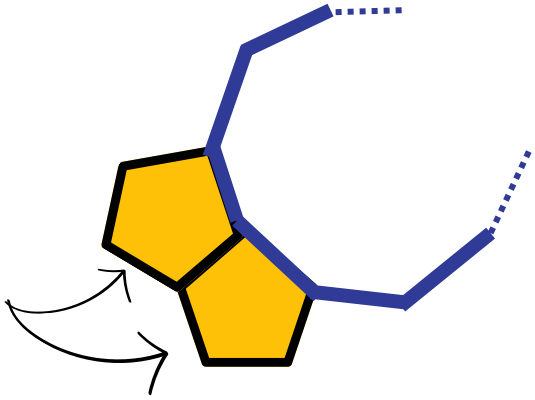
$$5x = 360°$$

$$x = \frac{360°}{72°}$$

$$x = 72°$$

Interior and Exterior Angles in Polygons

Example: Find out the value of number of sides of polygon -



Solution:

Step#1: The angle of a regular pentagon will be -

$$x = \frac{(5 - 2) \times 180°}{5}$$

$$x = \frac{540°}{5}$$

$$x = 108°$$

Step#2: Thus, the two angles together make

$$108° + 108° = 216°$$

And when the third angle (unknown polygon) is added to These two add up to 360°.

$$y + 216° = 360°$$

$$y = 360° - 216°$$

$$y = 144°$$

Step#3: As interior and exterior angles are supplementary -

$$EXT = 180° - 144°$$

$$EXT = 36°$$

Also number of sides will be then -

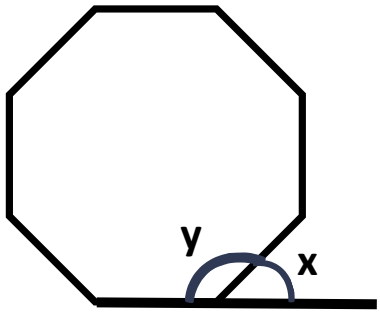
$$n = \frac{360°}{EXT} \qquad n = \frac{360°}{36°}$$

$$n = 10$$

Interior and Exterior Angles in Polygons

Solved Example

Example: Find out the value of exterior angle x and interior angle y of polygon



Solution:

Step#1: The polygon is an Octagon, and we can find the exterior angle by the formula -

$$n = \frac{360°}{EXT}$$

$$EXT = \frac{360°}{n}$$

$$EXT = \frac{360°}{8}$$

$$EXT = 45°$$

Step#2: Now as the Interior and Exterior angles are supplementary-

$$INT + EXT = 180°$$

$$INT = 180° - EXT$$

$$INT = 180° - 45°$$

$$INT = 135°$$

Step#3: Verification -

$$INT \times 8 = 1080°$$

$$(n - 2) \times 180° = 1080°$$

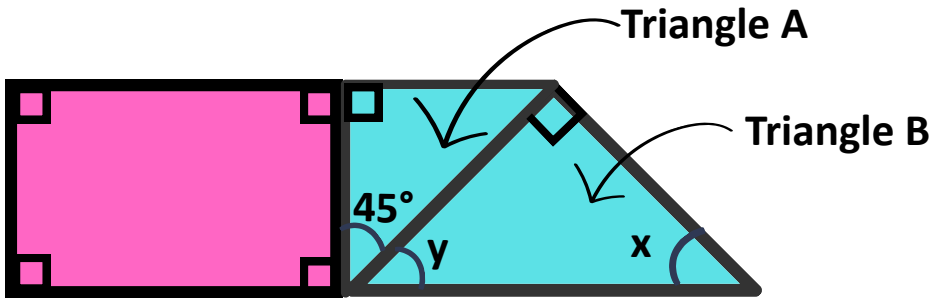
$$(n-2) = \frac{1080°}{180°}$$

$$n - 2 = 6$$

$n = 8$, hence proved.

Interior and Exterior Angles in Polygons

Example: Find the value of unknown angles-



Solution: The Interior angle of a rectangle is 90°, and the exterior angle will be -

$$INT + EXT = 180°$$

$$EXT = 180° - INT$$

$$EXT = 180° - 90°$$

$$EXT = 90°$$

Thus, in triangle A and B -

$$45° + \angle y = 90°$$

$$\angle y = 45°$$

Therefore, in triangle B -

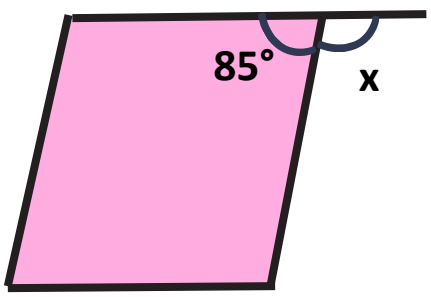
$$90° + \angle x + 45° = 180°$$

$$\angle x + 144° = 180°$$

$$\angle x = 180° - 144°$$

$$\angle x = 35°$$

Example: Find the value of unknown angle-



Solution: In the diagram, interior angle is 85° and x is unknown. As they are supplementary-

$$85° + \angle x = 180°$$

$$\angle x = 180° - 85°$$

$$\angle x = 95°$$