Conditional Probability – GCSE Maths

CONTENTS

1. Introduction

- **Conditional Probability** is the probability of an event occurring given that another event has already occurred.
- Studying of **Conditional Probability** is important because it helps us understand how the probability of an event changes when we know that another event has occurred.
- This concept is essential in real-world situations where outcomes are **not independent.**

2. What is Conditional Probability?

- We know, if one event depends upon the outcome of another event, the two events are **Dependent events**.
- A **Conditional Probability** is the probability of a dependent event in which probability of the second outcome depends on what has already happened in the first outcome.
- **Example:** If there is a bag with **red** and **blue** balls. Picking one ball out and don't put it back, then take another one, the chance of getting a red or blue ball on the second draw depends on what happened first.

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3. How to calculate Conditional Probability using Tree Diagrams?

A **Tree Diagram** can be used to solve Conditional Probability using dependent events

Steps to solve conditional Probability using Tree Diagram:

Steps#1: Draw the Branch and label the probabilities.

Steps#2: Add Dependent Branches

Steps#3: Apply the Condition

Steps#4: Find the Probability

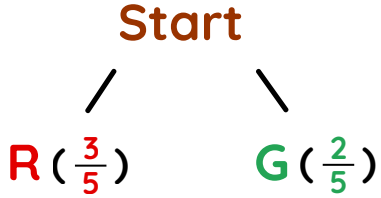
Example: Ivan has a Bag with 3 red and 2 green marbles. He picks 2 marbles without replacement. What's the probability the second marble is red given the first was green?

Solution:

Steps#1: Draw the Branch and label the probabilities.

Possible outcomes for first pick:

- **Total Marbles** = 5
- **Red (3 out of 5 marbles)** = $\frac{3}{5}$
- **Green (2 out of 5 marbles)** = $\frac{2}{5}$



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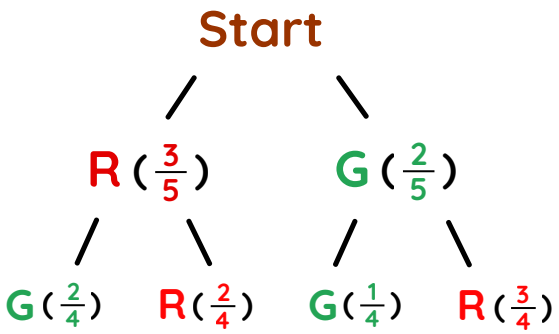
Steps#2: Add Dependent Branches

- **If first was Red:**
  - Remaining marbles: 2 red, 2 green
  - Next pick will be:

Red = $\frac{2}{4}$ **Green** = $\frac{2}{4}$

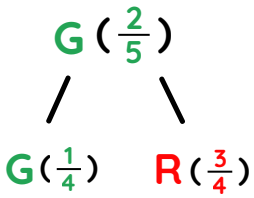
- **If first was Green:**
  - Remaining marbles: 3 red, 1 green
  - Next pick will be:

Red = $\frac{3}{4}$ **Green** = $\frac{1}{4}$



Steps#3: Apply the Condition

First marble was green, so only follow the green path.



Steps#4: Find the Probability

On the Green path, the chance the second marble is red is $\frac{3}{4}$

The probability the second marble is red is $\frac{3}{4}$

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4. Use Two-way Table to calculate Conditional Probability.

A **Two-way table** is a table shows how often different combinations of two events happen together.

Steps to solve conditional Probability using Two-way table:

Steps#1: Create the Two-Way Table

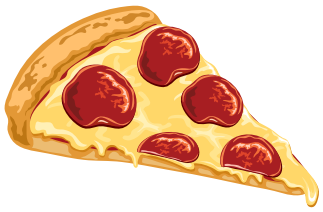
Steps#2: Apply the Condition

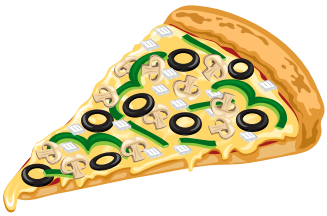
Steps#3: Find the Probability

Example: If a random cheesy pizza slice is picked from

- **6 pepperonis (with cheese)**
- **3 olive pizzas (with cheese)**

What's the probability it's pepperoni with cheese?





Solution:

Steps#1: Create the Two-Way Table

Total Pizza Slices = 9

Pizza Type	With Cheese	Probability
Pepperoni	6	$\frac{6}{9}$
Olive	3	$\frac{3}{9}$

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Steps#2: Apply the Condition

The Condition is pepperoni with cheese slice.

Steps#3: Find the Probability

Using the Table:

$$P(\text{pepperoni with cheese}) = \frac{6}{9}$$

The probability of the cheesy slice is pepperoni is $\frac{6}{9}$

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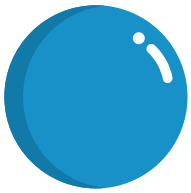
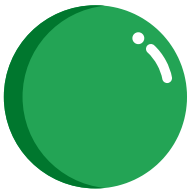
5. Solved Examples

Problem1: A bag contains:

4 Red



3 Blue



3 Green

You randomly pick one ball, don't put it back, then pick a second ball. What's the probability the second ball is red, given that the first ball was blue?

Solution:

Steps#1: Create the Two-Way Table

If the first ball was blue then,

- **Total number of balls left:** 10 - 1 = 9

Second Ball	Count Remaining	Probability
Red	4	$\frac{4}{9}$
Blue	2	$\frac{2}{9}$
Green	3	$\frac{3}{9}$

Steps#2: Apply the Condition

The condition is that if the first ball picked is blue, then the second ball is red.

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Steps#3: Find the Probability

Using the Table:

$$\text{P(second ball is red)} = \frac{4}{9}$$

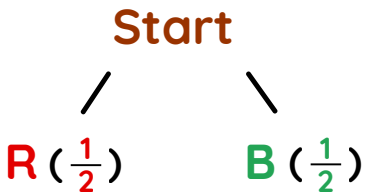
The probability of the second ball is red if the first was blue $\frac{4}{9}$

Problem2: A standard deck has 52 cards. You draw 2 cards without replacement. What's the probability the second card is red, given the first card was black?

Solution:

Steps#1: Draw the Branch and label the probabilities.

- **P(Black):** $\frac{26}{52} = \frac{1}{2}$
- **P(Red):** $\frac{26}{52} = \frac{1}{2}$



Steps#2: Add Dependent Branches

- **If first was Black:**
  - Remaining Cards: 26 red, 25 black
  - Next pick will be:

$$\text{P(Red)} = \frac{26}{51} \qquad \text{P(Black)} = \frac{25}{51}$$

- **If first was Red:**
  - Remaining Cards: 25 red, 26 black
  - Next pick will be:

$$\text{P(Red)} = \frac{25}{51} \qquad \text{P(Black)} = \frac{26}{51}$$

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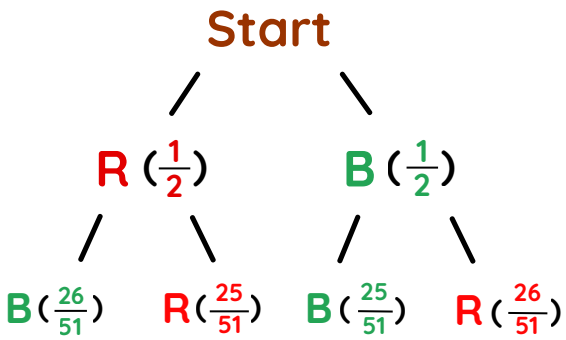
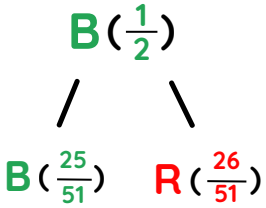


Steps#3: Apply the Condition

First Card was black, so only follow the black path.



Steps#4: Find the Probability

On the Black path, the chance the second card is red is $\frac{26}{51}$

The probability the second card is red is $\frac{26}{51}$