

Completing the Square – GCSE Maths

3. Steps to Solve Completing the Square.

- Completing the square is a method used to rewrite quadratic expressions in the form:

$$a(x - h)^2 + k$$

- Steps to convert Quadratic expression into the form of $a(x^2 - h) + k$ by Completing the Square:

Step1: Identify the Quadratic expression

$$ax^2 + bx + c$$

Step2: Solve for:

$$\frac{b}{2a}$$

Step3: Re-write the equation as,

$$a\left(x + \frac{b}{2a}\right)^2 + k$$

Step4: Plug $x = -\frac{b}{2a}$ to your original equation and solve for k.

Step5: Simplify it.

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Example: Factorize $2x^2+6x+9$ by completing the square.

Solution:

Step1: Identify the Quadratic expression.

Here:

- **a = 2** (coefficient of x^2)
- **b = 6** (coefficient of x)
- **c = 9** (constant term)

Step2: Solve for:

$$\frac{b}{2a} = \frac{6}{2(2)} = \frac{3}{2}$$

Step3: Re-write the equation as,

$$a\left(x + \frac{b}{2a}\right)^2 + k$$

$$2\left(x + \frac{3}{2}\right)^2 + k$$

Step4: Plug $x = -\frac{b}{2a}$ to your original equation and solve for k.

$$2\left(\frac{-3}{2}\right)^2 + 6\left(\frac{-3}{2}\right) + 9$$

$$2\left(\frac{9}{4}\right) - \frac{18}{2} + 9 = \frac{9}{2}$$

Step5: Simplify it.

$$2\left(x + \frac{3}{2}\right)^2 + \frac{9}{2}$$

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- We can also find the Turning point by Completing the Square

After finding the equation:

$$a\left(x + \frac{b}{2a}\right)^2 + k$$

Identify the Turning Point:

Comparing with,

$$y = a(x - h)^2 + k$$

where,

- **h** and **k** are the turning point (vertex) of the parabola.
- **a** determines the parabola's width and direction (upwards if $a > 0$, downwards if $a < 0$).

Example: Find the turning point of $y = 2(x-2)^2 - 3$

Solution:

Identify the Turning Point:

$$y = 2(x - 2)^2 - 3$$

- **h** = 2 (the x-coordinate of the vertex).
- **k** = -3 (the y-coordinate of the vertex).

The Turning point (vertex) of the parabola is (2, -3).

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4. Solved Examples

Problem1: Factorize x^2+6x+5 by completing the square.

Solution:

Step1: Identify the Quadratic expression.

Here:

- **a = 1** (coefficient of x^2)
- **b = 6** (coefficient of x)
- **c = 5** (constant term)

Step2: Solve for:

$$\frac{b}{2a} = \frac{6}{2(1)} = 3$$

Step3: Re-write the equation as,

$$a\left(x + \frac{b}{2a}\right)^2 + k$$

$$(x + 3)^2 + k$$

Step4: Plug $x = -\frac{b}{2a}$ to your original equation and solve for k.

$$(-3)^2 + 6(-3) + 5$$

$$9 - 18 + 5 = -4$$

Step5: Simplify it.

$$(x + 3)^2 - 4$$

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Problem2: Find the Turning Point of $y = 3(x - \frac{5}{3})^2 - \frac{4}{3}$

Solution:

Identify the Turning Point:

$$y = 3(x - \frac{5}{3})^2 - \frac{4}{3}$$

Where,

- $h = -\frac{5}{3}$ (the x-coordinate of the vertex).
- $k = -\frac{4}{3}$ (the y-coordinate of the vertex).

The Turning Point of parabola is $(-\frac{5}{3}, -\frac{4}{3})$

Problem3: Given the quadratic equation:

$$y = 2x^2 + 8x + 5$$

Rewrite the equation in completed square form and hence, find the turning point of the parabola.

Solution:

Step1: Identify the Quadratic expression.

Here:

- $a = 2$ (coefficient of x^2)
- $b = 8$ (coefficient of x)
- $c = 5$ (constant term)

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Step2: Solve for:

$$\frac{b}{2a} = \frac{8}{2(2)} = 2$$

Step3: Re-write the equation as,

$$a\left(x + \frac{b}{2a}\right)^2 + k$$

$$(x + 2)^2 + k$$

Step4: Plug $x = -\frac{b}{2a}$ to your original equation and solve for k.

$$2(-2)^2 + 8(-2) + 5$$

$$8 - 16 + 5 = -3$$

Step5: Simplify it.

$$2(x + 2)^2 - 3$$

Step6: Identify the Turning Point:

Vertex form:

$$a(x - h)^2 + k$$

Comparing with it,

$$h = -2 \quad \text{and} \quad k = -3$$