

FRACTIONS – GCSE MATHS

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1. Introduction

Fractions are a way to show parts of a whole. We use them when something is divided into equal pieces, and we want to talk about some of those pieces.

For example: if a pizza is cut into 8 equal slices and you eat 3, you've eaten $\frac{3}{8}$ of the pizza.

2. Types of Fractions

A fraction is a number that represents a part of a whole. It is written in the form $\frac{a}{b}$, where:

- **a** is the **numerator** – the number of parts you have.
- **b** is the **denominator** – the total number of equal parts the whole is divided into.

For example:

In $\frac{1}{2}$, the whole is divided into 2 parts, and we have 1 of them.

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Let us take an example:



a pizza of having 9 slices



one slice of pizza is in the form of fraction as: $\frac{1}{9}$

Now let us discuss different types of fractions:

- **Unit Fractions**

A unit fraction is a fraction in which the numerator is **always 1**, and the denominator is some **positive integer**. In other words, unit fractions are of the form $1/n$, where n is any natural number (**1, 2, 3, ...**)

For example: $\frac{1}{2}$, $\frac{1}{8}$, $\frac{1}{5}$

- **Improper Fractions**

An improper fraction is a fraction in which the numerator (the top number) is **greater than or equal** to the denominator (the bottom number). In other words, the **fraction** represents a value that is **equal to or greater than 1**.

For example: $\frac{5}{2}$, $\frac{3}{2}$, $\frac{6}{5}$

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- **Mixed Numbers**

A mixed number is a **combination of a whole number** and a proper fraction (a fraction where the numerator is **smaller** than the denominator). It is used to represent quantities **greater than 1** in a more intuitive way.

For example: $2\frac{1}{4}$, $5\frac{3}{5}$, $9\frac{3}{2}$

3. Converting into Forms

- **Converting Mixed fractions to Improper fractions.**

Steps:

1. Multiply whole number \times denominator.
2. Add numerator.
3. Write over the same denominator.

Problem: Convert $5\frac{1}{4}$ to an improper fraction.

Solution:

Step #1: Multiply whole number \times denominator.

$$5 \times 4 = 20$$

Step #2: Add numerator.

$$20 + 1 = 21$$

Step # Write over the same denominator.

$$\text{Result: } \frac{21}{4}$$

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- **Converting Decimals to Fractions.**

1. Terminating Decimals

A Terminating Decimal is a decimal number that ends after a finite number of digits.

Steps:

Step #1: Write the decimal as a fraction over 1

$$0.75 = \frac{0.75}{1}$$

Step #2: Multiply numerator and denominator by 10, 100, 1000, etc. to eliminate the decimal.

1 decimal place → × 10

2 decimal place → × 100

3 decimal place → × 1000

Step #3: Simplify the fraction by dividing numerator and denominator by their Greatest Common Divisor (GCD).

For example:

Problem: Convert 0.125 as a fraction

Solution:

Step #1: Write the decimal as a fraction over 1

$$0.125 = \frac{0.125}{1}$$

Step #2: Multiply numerator and denominator by 10, 100, 1000, etc. to eliminate the decimal.

3 decimal place → × 1000

Step #3: Simplify the fraction by dividing numerator and denominator by their Greatest Common Divisor (GCD).

$$\frac{0.125}{1000} = \frac{1}{8}$$

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2. Recurring Decimals

A Recurring Decimal is a decimal number in which one or more digits repeat infinitely after the decimal point.

Let x be the repeating decimal.

Steps:

Step #1: Multiply by 10^n (where n = number of repeating digits).

Step #2: **Subtract** the original equation to eliminate the repeating part.

Step #3: **Solve for x** and simplify.

For example:

Problem: Convert $0.\bar{7}$ to a fraction.

Solution:

Let $x = 0.\bar{7}$

Step #1: Multiply by 10 since 1 digit repeats itself

$$10x = 7.\bar{7}$$

Step #2: Subtract the original equation

$$10x - x = 7.\bar{7} - 0.\bar{7}$$

$$9x = 7$$

Step #3: Solve for x :

$$x = \frac{7}{9}$$

4. Reciprocals

Reciprocal is generally the "flip" of a fraction or the multiplicative inverse that results in 1 when multiplied by the original number.

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FRACTIONS: Flip the top and bottom of the fraction as

$$\frac{\overset{\text{orange}}{1}}{\underset{\text{green}}{3}} = \frac{\overset{\text{orange}}{4}}{\underset{\text{green}}{3}}$$

WHOLE NUMBER: It is 1 divided by that number.

$$5 = \frac{\overset{\text{orange}}{1}}{\underset{\text{green}}{5}}$$

MIXED NUMBERS: Firstly convert into improper fraction and then flip the equation

$$2\frac{\overset{\text{orange}}{3}}{\underset{\text{green}}{4}} = \frac{\overset{\text{blue}}{11}}{\underset{\text{green}}{4}} = \frac{\overset{\text{orange}}{4}}{\underset{\text{green}}{11}}$$

5. Operations with Fractions

1. Addition and Subtraction

Steps:

Step #1: Make the bottom numbers the same

Step #2: Add or subtract the top numbers

Step #3: Convert back to a mixed number if needed

2. Multiplication

Steps:

Step #1: Multiply top numbers together.

Step #2: Multiply bottom numbers together.

Step #3: Simplify the result.

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3. Division

Steps:

Step #1: Keep the first fraction the same.

Step #2: Change the division sign to a multiplication sign.

Step #3: Flip the second fraction over to write the reciprocal.

Solved Examples:

Problem: Convert $3/4 + 7/2$ as a fraction

Solution:

Step #1: Make the bottom numbers the same

$$\begin{aligned} &= \frac{3}{4} + \frac{7}{2} \\ &= \frac{3+14}{8} \end{aligned}$$

Step #2: Add the top numbers

$$= \frac{17}{4}$$

Step #3: Convert back to a Mixed Fraction

$$= 3\frac{5}{4}$$

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Problem: Convert $1/3 \times 3/5$ as a fraction

Solution:

Step #1: Multiply top numbers together.

$$= \frac{1}{3} \times \frac{3}{5}$$

Step #2: Multiply bottom numbers together.

$$= \frac{1 \times 3}{3 \times 5}$$

Step #3: Simplify the result

$$\frac{3}{15} = \frac{1}{5}$$

Problem: Convert $3/4 \div 7/2$ as a fraction

Solution:

Step #1: Keep the first fraction same and change the divide sign to multiplication sign and reciprocate the second fraction.

$$= \frac{3}{4} \times \frac{2}{7}$$

Step #2: Multiply bottom and top numbers together.

$$= \frac{6}{28}$$

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6. Solved Examples

Problem: A builder completed $\frac{5}{8}$ of a wall on Monday and $\frac{1}{4}$ on Tuesday.
What fraction of the wall is left to complete?

Solution:

Step #1: Write down the given information

On Monday, fraction of wall gets completed:- $\frac{5}{8}$

On Tuesday, fraction of wall gets completed:- $\frac{1}{4}$

Step #2: Simplify to make a common denominator

We know that:

$$\frac{1}{4} = \frac{2}{8}$$

Step #3: Calculate the final result by applying favorable operations

The total amount of work that has been completed: $\frac{5+2}{4} = \frac{7}{8}$

Fraction of wall that is left to complete: $1 - \frac{7}{8}$

$$= \frac{1}{8}$$

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Problem: A box has 24 pencils. $\frac{1}{3}$ of them are red, and $\frac{1}{4}$ are blue. How many pencils are neither red nor blue?

Solution:

Step #1: Write down the given information

Red pencils :- $\frac{1}{3} \times \frac{8}{8} = \frac{8}{24}$

Blue pencil :- $\frac{1}{4} \times \frac{6}{6} = \frac{6}{24}$

Step #2: Simplify to make a common denominator

We know that:

$$\frac{8 + 6}{24} = \frac{14}{24}$$

Step #3: Calculate the final result by applying favorable operations

The total number of pencils that is either red or blue : $\frac{14}{24}$

Pencils that are neither red nor blue: $1 - \frac{14}{24}$

$$\frac{10}{24}$$