Law of Sine and Cosine Rule – GCSE Maths

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1. Introduction

- **Laws of Sine and Cosine** are trigonometric formulas used to solve triangles when certain information is given.
- They are especially useful for **non-right triangles.**
- These laws are fundamental in trigonometry and have applications in physics, engineering, and navigation.

2. What is the Sine rule?

- The **Sine Rule** is a fundamental trigonometric formula that relates the sides of a triangle to the sines of their opposite angles.
- Mathematically,

  For any triangle with sides **a**, **b** and **c** opposite angles **A**, **B** and **C** respectively, for finding missing side:

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

  Alternatively, it can be written as for finding missing angle:

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

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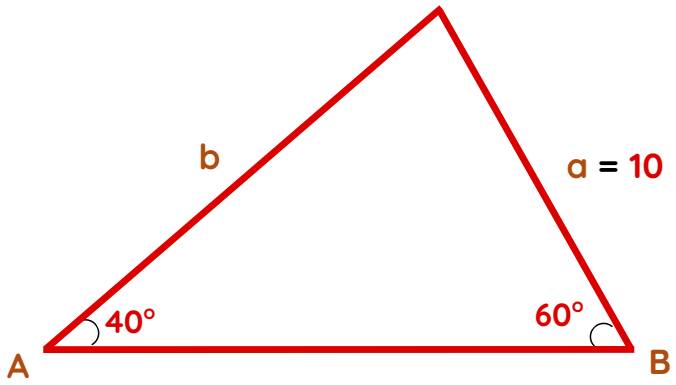
Where:

- a, b and c are the lengths of the sides of the triangle
- A, B and C are the angles opposite those sides

Example: **A** = 40°, **B** = 60° and side **a** = 10 cm. Find side b.



Solution:

Use the formula,

$$\frac{a}{\sin A} = \frac{b}{\sin B}$$

Put the values,

$$\frac{10}{\sin 40^\circ} = \frac{b}{\sin 60^\circ}$$

$$b = \frac{10 \times \sin 60^\circ}{\sin 40^\circ}$$

Now calculate using a calculator,

$$b = \frac{10 \times 0.866}{0.642}$$

$$b \approx 13.5 \text{ cm}$$

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3. What is the Cosine rule?

- **The Cosine Rule** is also a trigonometric formula used to find a side or angle in a triangle.
- It works for **any triangle** whether it's acute, obtuse, or right-angled.
- Mathematically,

  For any triangle with sides **a**, **b** and **c** opposite angles **A**, **B** and **C** opposite those sides:

$$a^2 = b^2 + c^2 - 2bc\,\text{Cos}(A)$$

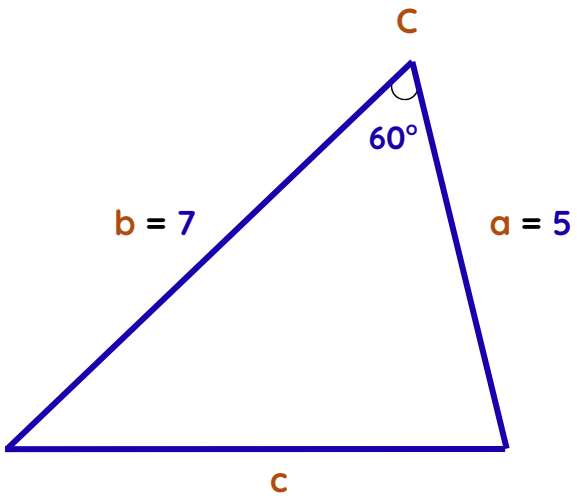
  If you know all three sides, then we can find an angle using this rearranged version of the cosine rule:

$$\text{Cos}(A) = \frac{b^2 + c^2 - a^2}{2bc}$$

  Where:

  - a, b and c are the lengths of the sides of the triangle.
  - A, B and C are the angles opposite those sides.

Example: Side **a** = 5cm, side **b** = 7 cm, angle **C** = 60°. Find side **c.**



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Solution:

Use the formula,

$$c^2 = a^2 + b^2 - 2ab\ Cos(C)$$

Put the values,

$$c^2 = 5^2 + 7^2 - 2(5)(7)\ Cos(60°)$$

$$c^2 = 25 + 49 - 70(0.5)$$

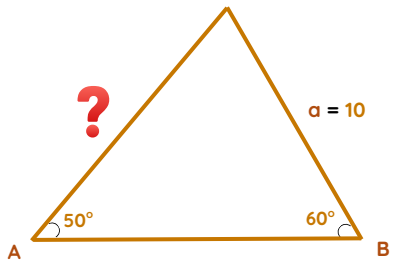
$$c^2 = 74 - 35 = 39$$

$$c = \sqrt{39}$$

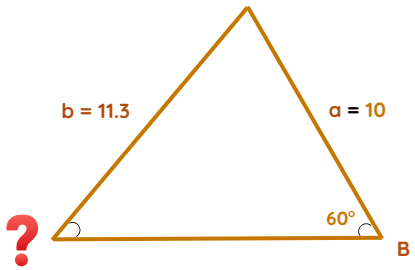
$$c \approx 6.24\ cm$$

4. How to find missing side and angle?

- The Sine Rule or the Cosine Rule, both are used to find the missing side or missing angle depending on what information is given in the question.
- **Use the Sine Rule:**
- If we know the 2 angles and one side, then we use it to find another side



- If we know the 2 sides and one non-included angle, then we use it to find the other angle.



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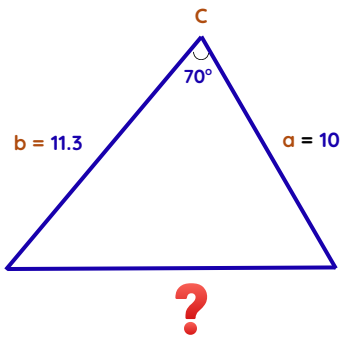
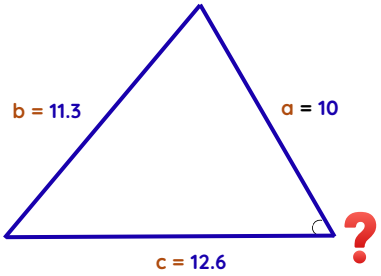
- **Use the Cosine Rule:**
- If we know the 2 sides and one included angle, then we use it to find third side.



- If we know all the three sides, then we use it to find any angle.



Steps to find the missing side or angle:

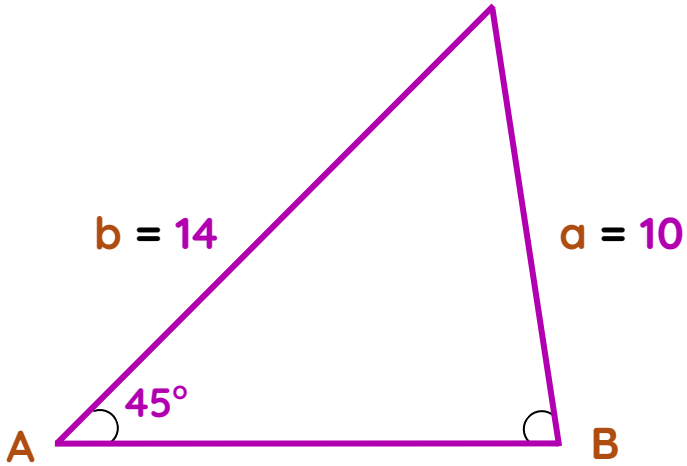
Step#1: Identify the known values.

Step#2: Write the formula based on the side or angle you're finding.

Step#3: Plug the values.

Step#4: Solve for the missing value.

Example: In Triangle **ABC**, **Side a** = 10cm, **Side b** = 14cm and **Angle A** = 45°. Find angle **B**.



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Solution:

Step#1: Identify the known values.

Given:

- **Side a** = 10cm
- **Side b** = 14cm
- **Angle A** = 45°

Step#2: Using the formula,

$$\frac{\sin B}{b} = \frac{\sin A}{a}$$

Step#3: Plug the values,

$$\frac{\sin B}{14} = \frac{\sin 45°}{10}$$

Step#4: Solve for the missing angle.

$$\sin B = \frac{14 \times \sin 45°}{10}$$

$$\sin B = \frac{14 \times 0.7071}{10}$$

$$\sin B \approx 0.9899$$

$$B = \sin^{-1}(0.9899)$$

$$B \approx 81.6°$$

The Missing angle of **B ≈ 81.6°**

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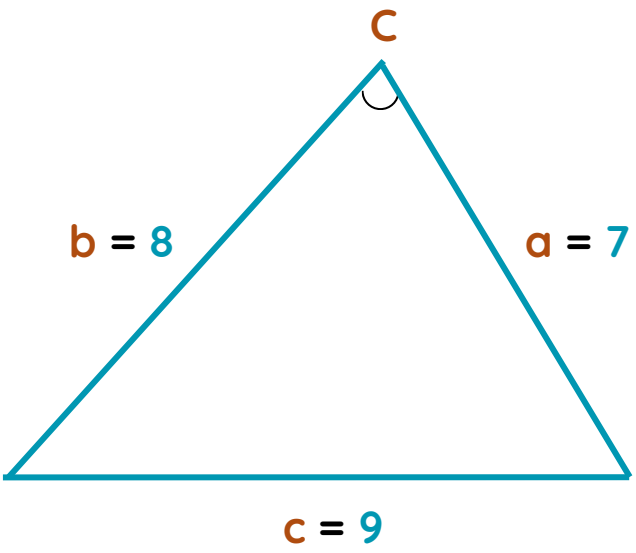
5. Solved Examples

Problem1: In Triangle **ABC**, **Side a** = 7cm, **Side b** = 8cm and **Side c** = 9cm. Find angle **C**.



Solution:

Step#1: Identify the known values.

Given:

- **Side a** = 7cm
- **Side b** = 8cm
- **Side c** = 9cm

Step#2: Using the formula,

$$\text{Cos}(C) = \frac{a^2 + b^2 - c^2}{2ab}$$

Step#3: Plug the values,

$$\text{Cos}(C) = \frac{7^2 + 8^2 - 9^2}{2(7)(8)}$$

Step#4: Solve for the missing angle.

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$$\text{Cos}(C) = \frac{49 + 64 - 81}{112}$$

$$\text{Cos}(C) = \frac{32}{112}$$

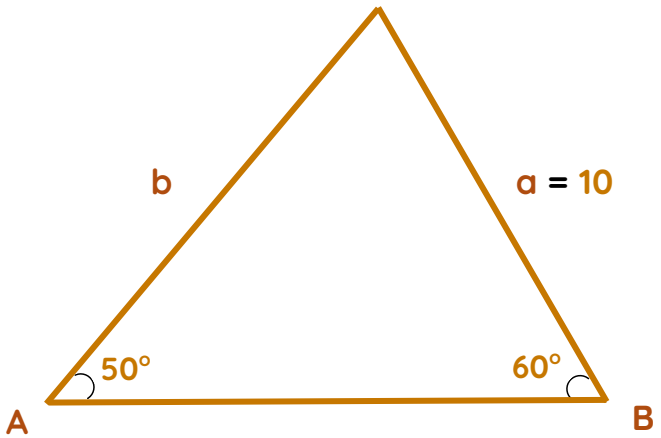
$$\text{Cos}(C) \approx 0.2857$$

$$C = \cos^{-1}(0.2857)$$

$$C \approx 73.4^\circ$$

The Missing angle of **C ≈ 73.4°**

Problem2: In Triangle **ABC**, **Angle A** = 50°, **Angle B** = 60° and **Side a** = 10cm. Find side **b**.



Solution:

Step#1: Identify the known values.

Given:

- **Side a** = 10cm
- **Angle A** = 50°
- **Angle B** = 60°

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Step#2: Using the formula,

$$\frac{a}{\sin A} = \frac{b}{\sin B}$$

Step#3: Plug the values,

$$\frac{10}{\sin 50^\circ} = \frac{b}{\sin 60^\circ}$$

Step#4: Solve for the missing angle.

$$b = \frac{10 \times \sin 60^\circ}{\sin 50^\circ}$$

$$b = \frac{10 \times 0.866}{0.766}$$

$$b \approx 11.31 \text{ cm}$$

The Missing side of **b ≈ 11.31 cm**