Independent Events – GCSE Maths

CONTENTS

1. Introduction

- In Probability Theory, **Independent events** are the important concepts that describe different relationships between events.
- It describes how the occurrence of one event affects the probability of another event.

2. What are Independent events?

- Two events are **Independent** if one event does not affect the probability of the other.
- **For example:** Flipping heads with a coin has no effect on rolling an even number with some dice, so they are **Independent events**.
- If the occurrence of one does affect the probability of the other, then it is called **Dependent Events**.
- Mathematically,

Two events A and B are Independent if:

$$P(A \cap B) = P(A) \times P(B) \quad \text{or} \quad P(A \text{ and } B) = P(A) \times P(B)$$

Two events A and B are Dependent if:

$$P(A \cap B) \neq P(A) \times P(B)$$

Independent events – GCSE Maths

3. Steps to solve the Independent events

- To determine whether two events are independent and calculate their probabilities, follow these steps:

  Step#1: Identify Events

  Step#2: Use Formula

  Step#3: Calculate the Probability

Example: If we flip a fair coin and roll a fair 6-sided die. What's the probability of getting Tails on the coin and an even number on the die?

Solution:

Step#1: Identify Events

- **Event A:** Coin lands Tails: **P(A)** = $\dfrac{1}{2}$
- **Event B:** Die shows even number (2, 4 and 6): **P(B)** = $\dfrac{3}{6}$ = $\dfrac{1}{2}$

Step#2: Use Formula

$$P(A \cap B) = P(A) \times P(B)$$

Step#3: Calculate the Probability

Put the Values in formula,

$$P(A \cap B) = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$$

Final Answer is $\dfrac{1}{4}$

Independent events – GCSE Maths

4. Solved Example

Problem1: A coin is tossed and a die is rolled. What is the probability of getting Heads on the coin and a 4 on the die?

Solution:

Step#1: Identify Events

- **Event A:** Getting Heads on the coin: **P(A)** = $\dfrac{1}{2}$
- **Event B:** Getting a 4 on the die: **P(B)** = $\dfrac{1}{6}$

Step#2: Use Formula

$$P(A \cap B) = P(A) \times P(B)$$

Step#3: Calculate the Probability
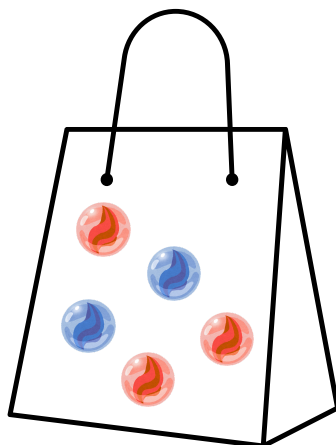
Put the Values in formula,

$$P(A \cap B) = \dfrac{1}{2} \times \dfrac{1}{6} = \dfrac{1}{12}$$

Final Answer is $\dfrac{1}{12}$

Independent events – GCSE Maths

Problem2: A bag contains **3 red marbles** and **2 blue marbles**. You randomly pick a marble, put it back, and then draw again. What is the probability of getting:

1. A red marble first
2. A blue marble second

Solution:

Step#1: Identify Events

Total marbles = **3 red** + **2 blue** = **5 marbles.**

- **Event A:** First draw is red: **P(A)** = $\dfrac{3}{5}$
- **Event B:** Second Draw is Blue: **P(B)** = $\dfrac{2}{5}$

Step#2: Use Formula

$$P(A \cap B) = P(A) \times P(B)$$
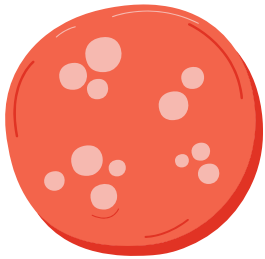
Step#3: Calculate the Probability

Put the Values in formula,

$$P(A \cap B) = \frac{3}{5} \times \frac{2}{5} = \frac{6}{25}$$

Final Answer is $\dfrac{6}{25}$
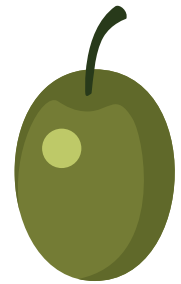
Independent events – GCSE Maths

Problem3: A restaurant serves pizza with 3 topping choices: **Pepperoni (P)**, **Mushrooms (M)**, and **Olives (O)**. Each topping has an independent probability of being selected by a customer:

P(P) = 0.6   **P(M)** = 0.4   **P(O)** = 0.3

What is the probability that a customer orders a pizza with both **Pepperoni** and **Mushrooms**.

 Solution:

 Step#1: Identify Events

- **Event A:** Customer selects Pepperoni: **P(A)** = **0.6**
- **Event B:** Customer selects Mushrooms: **P(B)** = **0.4**

 Step#2: Use Formula

$$P(A \cap B) = P(A) \times P(B)$$

 Step#3: Calculate the Probability

Put the Values in formula,

$$P(A \cap B) = 0.6 \times 0.4 = 0.24$$

Final Answer is **0.24**

Independent events – GCSE Maths

Problem4: If roll a fair 6-sided die twice, what is the probability of getting a 5 on the first roll and a 2 on the second roll?

Solution:

Step#1: Identify Events

- **Event A:** First die roll: **P(5)** = $\frac{1}{6}$
- **Event B:** Second die roll: **P(2)** = $\frac{1}{6}$

Step#2: Use Formula

$$P(A \cap B) = P(A) \times P(B)$$

Step#3: Calculate the Probability

Put the Values in formula,

$$P(A \cap B) = \frac{1}{6} \times \frac{1}{6} = \frac{1}{36}$$

Final Answer is $\frac{1}{36}$