Calculate Area using Sine Rule – GCSE Maths

CONTENTS

1. Introduction

- **Law of Sines** are trigonometric formulas used to solve any triangles when certain information is given.
- This law is used to find unknown sides or angles in non-right-angled triangles, it can also be applied to calculate **the area of a triangle** when certain information is given.
- It is a fundamental tool used to solve real-world problems involving triangles.

2. What is the Sine rule?

- The Sine Rule states that, in any triangle, the ratio of the length of a side to the sine of its opposite angle is the same for all three sides.
- The Sine Law is expressed as:

  For any triangle with sides **a**, **b** and **c** opposite angles **A**, **B** and **C** respectively, for finding missing side:

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

Note: *Learn more about sine rule*

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3. Sine Rule for Calculating the Area of a Triangle

- The Sine Rule is not just used for solving sides and angles, but it is also helpful to calculate the area of triangle especially when height is unknown.
- Mathematically,
  For any triangle with sides a, b, c and opposite angles A, B and C:

$$\text{Area} = \frac{1}{2}ab\sin(C)$$

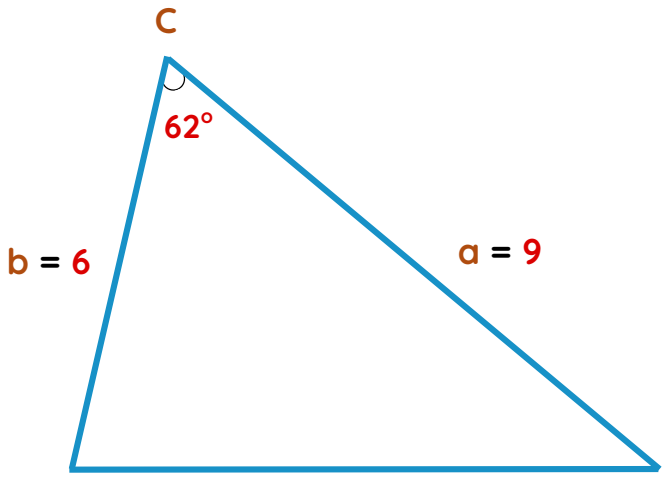
Where:

- a and b are two known sides.
- C is the angle between them (included angle).

It can also use as:

$$\text{Area} = \frac{1}{2}bc\sin(A) \quad \text{or} \quad \text{Area} = \frac{1}{2}ac\sin(B)$$

Example: A triangle has sides **a = 9cm**, **b = 6cm**, and the included angle **C = 62°**. Find its area.



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Solution:

Using the formula,

$$\text{Area} = \frac{1}{2}\,ab\,\sin(C)$$

Plug the values and solve,

$$\text{Area} = \frac{1}{2}(9)(6)\,\sin(62^\circ)$$

$$\text{Area} = \frac{1}{2}\,54 \times 0.8829$$

$$\text{Area} = 27 \times 0.8829$$

$$\text{Area} = 23.8383\ \text{cm}^2$$

4. How to Calculate area of triangle using sine rule?

- We can calculate the area of any triangle using the sine rule, based area formula, especially when we know two sides and the included angle between them.
- **Steps to calculate the area of triangle:**

  Step#1: Identify the known values

  Step#2: Use the formula based on information.

  Step#3: Plug the values in the formula.

  Step#4: Calculate the area

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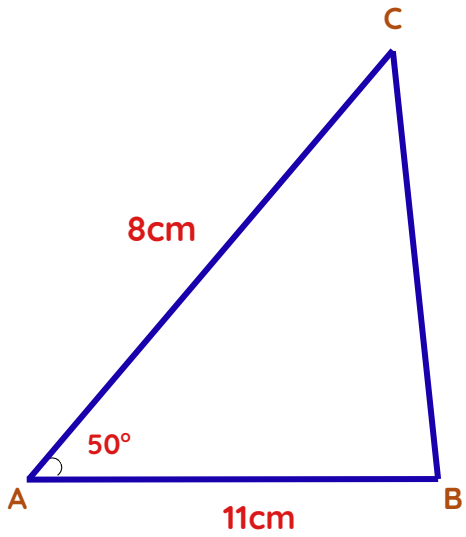
Example: In Triangle **ABC**, side **AB = 11 cm**, side **AC = 8 cm** and the angle between them ∠**A = 50**°.Find the area of triangle ABC.



Solution:

Step#1: Identify the known values

Given:

- Side AB = 11 cm
- Side AC = 8 cm
- Included angle ∠A = 50°

Step#2: Use the formula

$$\text{Area} = \frac{1}{2}(AB)(AC)\sin(A)$$

Step#3: Plug the values in the formula.

$$\text{Area} = \frac{1}{2}(11)(8)\sin(50^\circ)$$

Step#4: Calculate the area

$$\text{Area} = \frac{1}{2}88 \times 0.766$$

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$$\text{Area} = \frac{1}{2} 88 \times 0.766$$

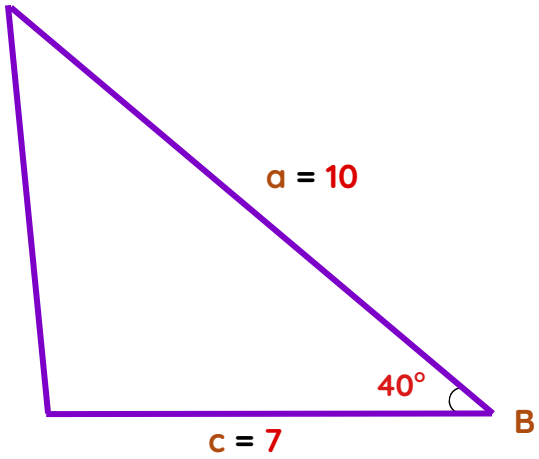
$$\text{Area} = 44 \times 0.766$$

$$\text{Area} = 33.7 \text{ cm}^2$$

Area of ABC triangle is **33.7cm^2**.

5. Solved Examples

Problem1: In Triangle, sides **a = 10 cm**, side **c = 7 cm** and the angle **B = 40°**. Find the area of triangle.



Solution:

Step#1: Identify the known values

Given:

- Side a = 10 cm
- Side c = 7 cm
- Angle B = 40°

Step#2: Use the formula

$$\text{Area} = \frac{1}{2} ab \sin(C)$$

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Step#3: Plug the values in the formula.

$$\text{Area} = \frac{1}{2}(10)(7)\sin(40^\circ)$$

Step#4: Calculate the area

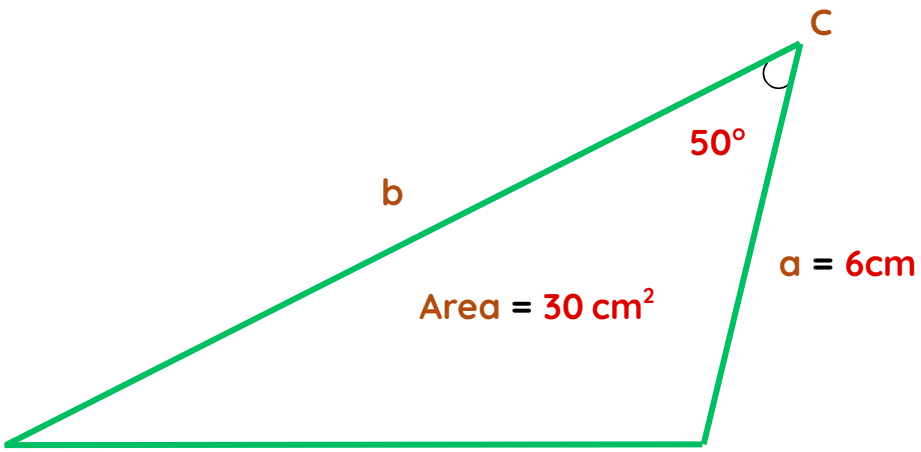
$$\text{Area} = \frac{1}{2}\ 70 \times 0.6428$$

$$\text{Area} = 35 \times 0.6428$$

$$\text{Area} = 22.50\ \text{cm}^2$$

Area of triangle is **22.50cm^2**.

Problem2: The area of a triangle is **30 cm^2**. One side **a = 6 cm**, and the included angle **C = 50°**.Find the other side b.



Solution:

Step#1: Identify the known values

Given:

- a = 6 cm
- Area of Triangle = 30 cm^2
- Angle c = 50°

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Step#2: Use the formula

$$\text{Area} = \frac{1}{2}\, ab \sin(C)$$

Rearrange it,

$$b = \frac{2 \times \text{Area}}{a \times \sin(C)}$$

Step#3: Plug the values in the formula.

$$b = \frac{2 \times 30}{6 \times \sin(50^\circ)}$$

Step#4: Calculate the area

$$b = \frac{60}{6 \times 0.766}$$

$$b = \frac{60}{4.596}$$

$$b = 13.05 \text{ cm}$$

Final answer is **13.05 cm**