Sectors of a Circle - GCSE Maths
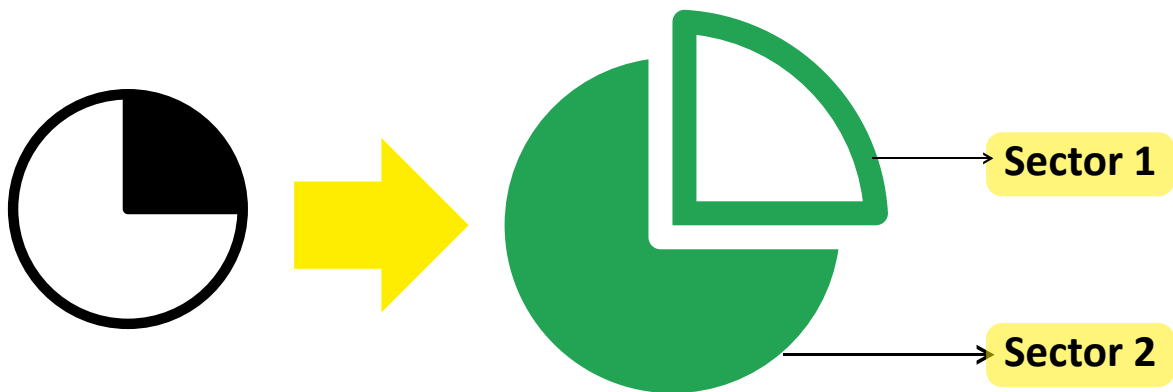
Contents

1. Introduction

Circle is very important 2-Dimensional shape in geometry. Sector is a part of circle. Sector's important characteristics are -

- A Sector of circle is the portion made by **two radii and the arc** connecting the ends of those radii. The shape can be viewed as a pizza slice.
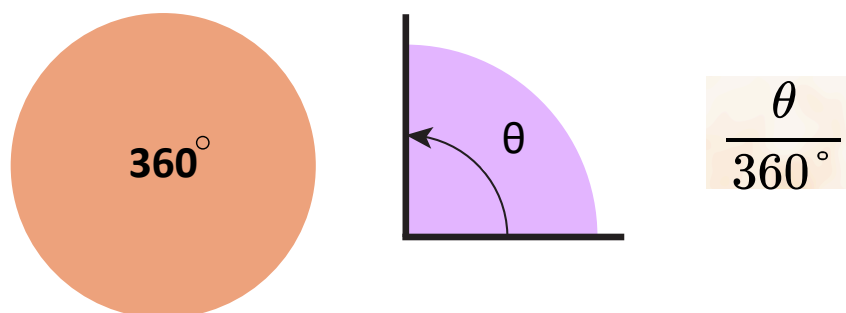
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- The following are two sectors of same circle -

Radius

Arc

θ

θ

Radius

Radius

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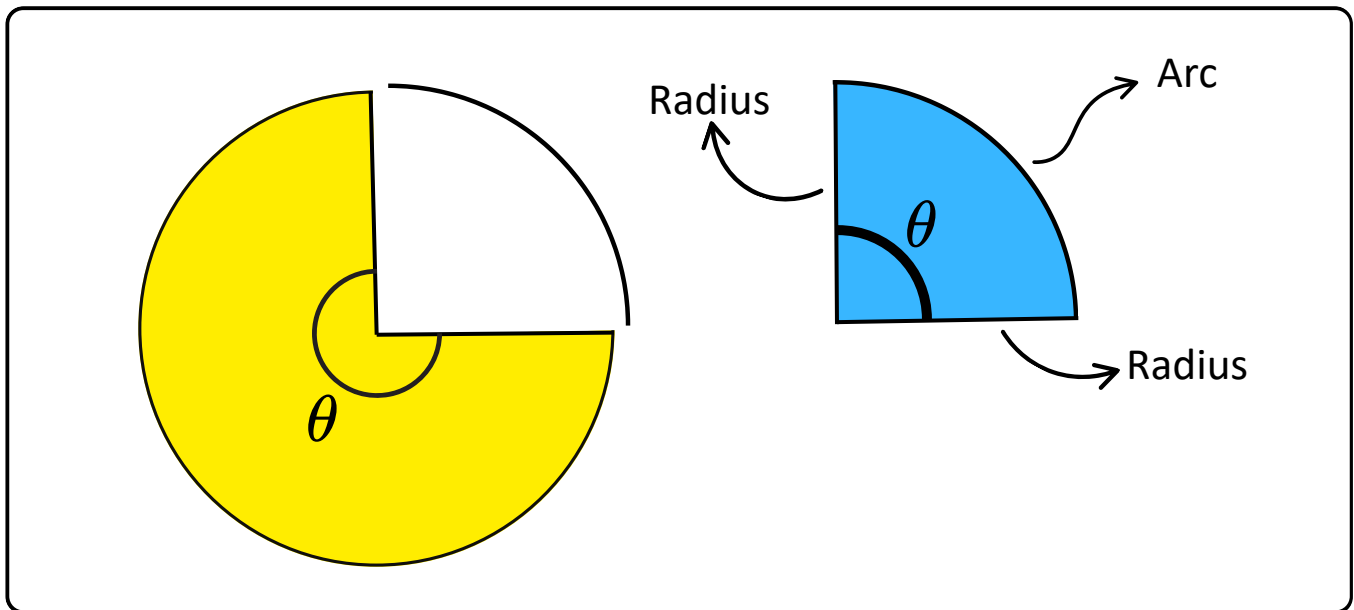
2. Area of a Sector

- A circle have a complete angle but Sector have a portion of it and we can represent the portion by writing it in fraction -
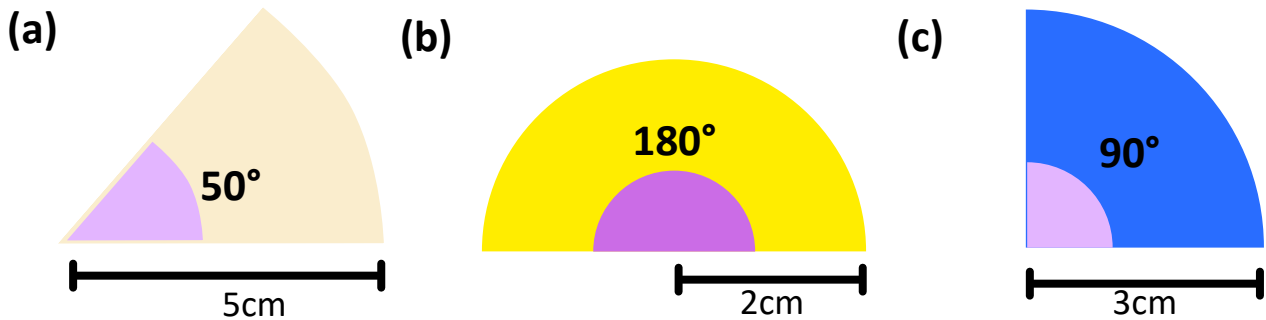
360°

θ

$$\frac{\theta}{360°}$$

- The circle have an area equal to $\pi(r)^2$, So Sector will have area equal to -

Angle

$$\frac{\theta}{360°} \times \pi \times (r)^2$$

Radius

3. Solved Examples of Area of Sector

Example: Find the area of following sectors -

(a)

50°

5cm

(b)

180°

2cm

(c)

90°

3cm

Solution:

- Formula for finding area of sector -

$$\frac{\theta}{360°} \times \pi \times (r)^2$$

(a) The area of sector with angle 50° and radius 5cm -

$$\text{Area} = [\frac{50°}{360°} \times \pi \times (5)^2]$$

$$\text{Area} = (\frac{5}{36} \times 3.14 \times 25) \text{ cm}^2 \quad (\text{using } \pi = 3.14)$$

$$\text{Area} = (0.436 \times 25) \text{ cm}^2$$

$$\text{Area} = 10.91 \text{ cm}^2$$

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(b) The area of sector with angle 180° and radius 2cm -

$$Area = [\frac{180°}{360°} \times \pi \times (2)^2] \ cm^2$$

$$Area = (\frac{1}{2} \times 3.14 \times 4) \ cm^2 \quad (using \ \pi = 3.14)$$

$$Area = (1.57 \times 4) \ cm^2$$

$$Area = 6.28 \ cm^2$$

(c) The area of sector with angle 90° and radius 3cm -

$$Area = [\frac{90°}{360°} \times \pi \times (3)^2] \ cm^2$$

$$Area = (\frac{1}{4} \times 3.14 \times 9) \ cm^2 \quad (using \ \pi = 3.14)$$

$$Area = (0.785 \times 9) \ cm^2$$

$$Area = 7.065 \ cm^2$$

4. Arc length of a Sector

- Sector is a part of circle similarly the arc of sector is part of the circumference of whole circle -

Sector part = $\dfrac{\theta}{360°}$

Circumference = **2πr**
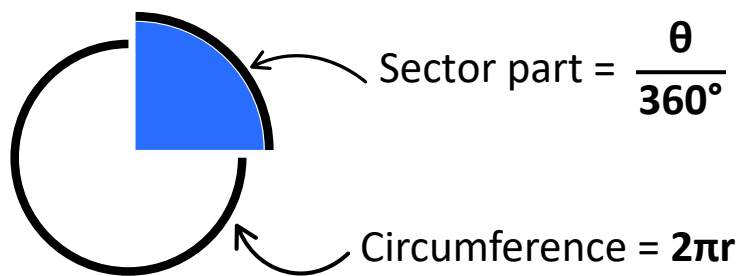
- Thus, the formula of Arc length of Sector part is -

Angle $\dfrac{\theta}{360°}$ X **2πr**

2r = Diameter

Example: Find out the arc length of the sector given in diagram -

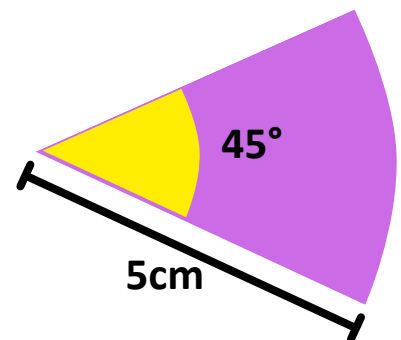**Solution:** The arc length of sector of circle with radius 5cm and angle 45° -

Arc Length = $\left(\dfrac{\theta}{360°} \text{ X } 2πr \right)$ cm

Arc Length = $\left(\dfrac{45°}{360°} \text{ X } 2 \text{ X } 3.14 \text{ X } 5 \right)$ cm

Arc Length = (0.785 X 3.14 X 5) cm
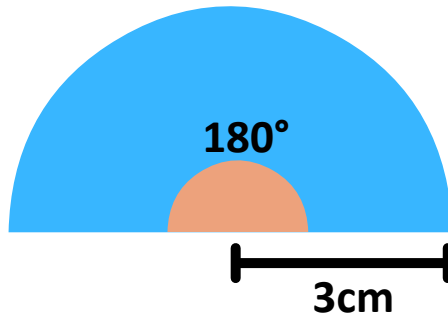
Arc Length = (2.46 X 5) cm

Arc Length = 12.3 cm

45°

5cm

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5. Solved Examples of Arc Length

Example: Work out the arc length of the Sectors -

(a)

180°

3cm

(b)

90°

4cm
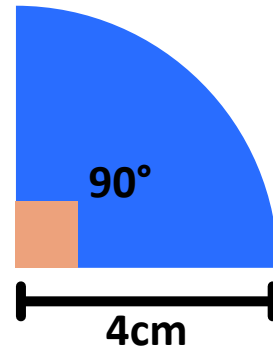
Solution: The formula of finding arc length -

$$\text{Arc Length} = \left(\frac{\theta}{360°} \times 2\pi r \right) \text{cm}$$

(a) Arc length of sector with radius 3cm and angle 180 degree

$$\text{Arc Length} = \left(\frac{180°}{360°} \times 2 \times 3.14 \times 3 \right) \text{cm}$$

$$\text{Arc Length} = \left(\frac{1}{2} \times 2 \times 3.14 \times 3 \right) \text{cm}$$

$$\text{Arc Length} = 9.42 \text{ cm}$$

(b) Arc length of sector with radius 4cm and angle 90 degree

$$\text{Arc Length} = \left(\frac{90°}{360°} \times 2 \times 3.14 \times 4 \right) \text{cm}$$

$$\text{Arc Length} = \left(\frac{1}{4} \times 2 \times 3.14 \times 4 \right) \text{cm}$$

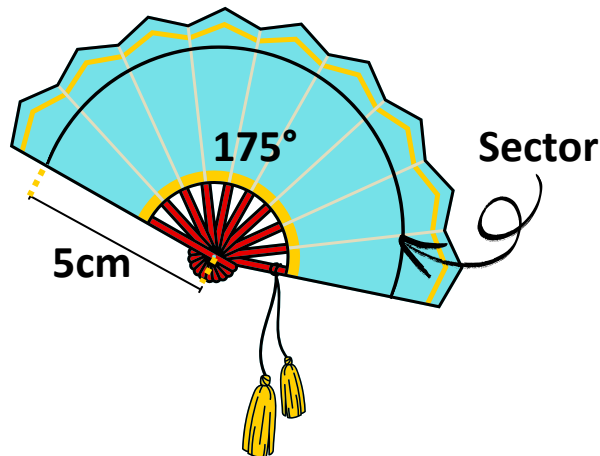$$\text{Arc Length} = 6.82 \text{ cm}$$

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Example: Work out the Length of arc of sector and area of Sector the fan represents -

175°

Sector

5cm

Solution: The formula to find the area of sector and Arc Length -

$$\text{Area} = \frac{\theta}{360°} \text{ X } \pi \text{ X } (r)^2 \qquad \text{Arc Length} = \frac{\theta}{360°} \text{ X } 2\pi r$$

The area of the sector shown in diagram -

$$\text{Area } = \frac{\theta}{360°} \text{ X } \pi \text{ X } (r)^2$$

$$\text{Area } = [\frac{175°}{360°} \text{ X } 3.14 \text{ X } (5)^2] \text{ cm}^2$$

$$\text{Area } = (\frac{175°}{360°} \text{ X } 3.14 \text{ X } 25) \text{ cm}^2$$

$$\text{Area } = 0.486 \text{ X } 3.14 \text{ X } 25 = 38.151 \text{ cm}^2$$

The arc length of sector shown in diagram -

$$\text{Area } = \frac{\theta}{360°} \text{ X } 2\pi r$$

$$\text{Area } = (\frac{175°}{360°} \text{ X } 2 \text{ X } 3.14 \text{ X } 5) \text{ cm}$$

$$\text{Area } = (\frac{175°}{360°} \text{ X } 3.14 \text{ X } 10) \text{ cm}$$

$$\text{Area } = 1.52 \text{ X } 10 = 15.2 \text{ cm}$$