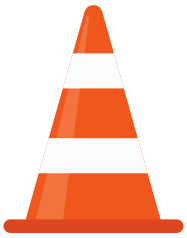
Pyramid and Cones – GCSE Maths

CONTENTS:

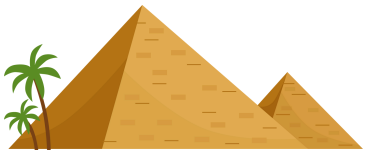
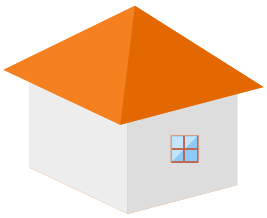
1. Introduction

- **Cones** and **Pyramids** are three-dimensional (3D) geometric shapes that play essential role in mathematics, architecture, engineering, and everyday life.
- Both shapes have a base and an apex, but they differ in structure and properties.
- Real life Examples:

CONE:


PYRAMIDS:

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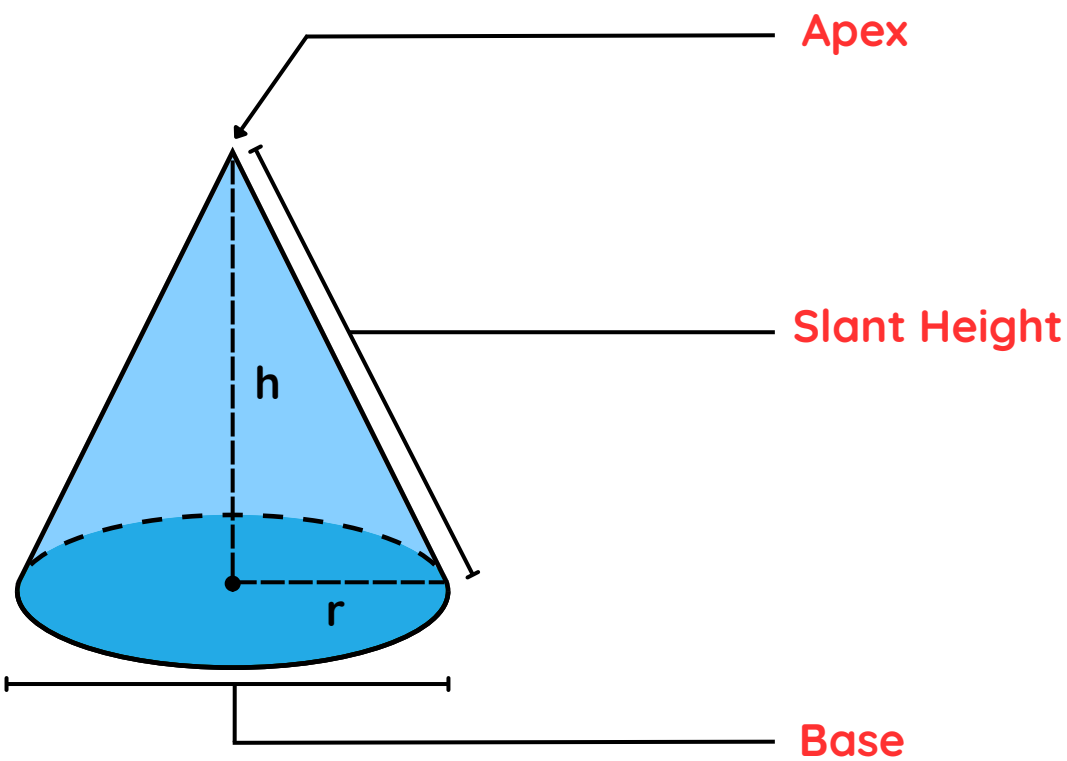
2. What is a Cone?

- A cone is a solid figure with a circular base that curves upward to meet at a single vertex, forming a pointed tip.

- Key Features of a Cone:

  - **Base -** A circular flat surface.
  - **Apex -** The pointed top where all the sides meet.
  - **Height (h) -** The perpendicular distance from the base to the apex.
  - **Slant Height (l) -** The distance from the apex to any point on the edge of the base.
  - **Radius (r) -** The distance from the center of the base to its edge.



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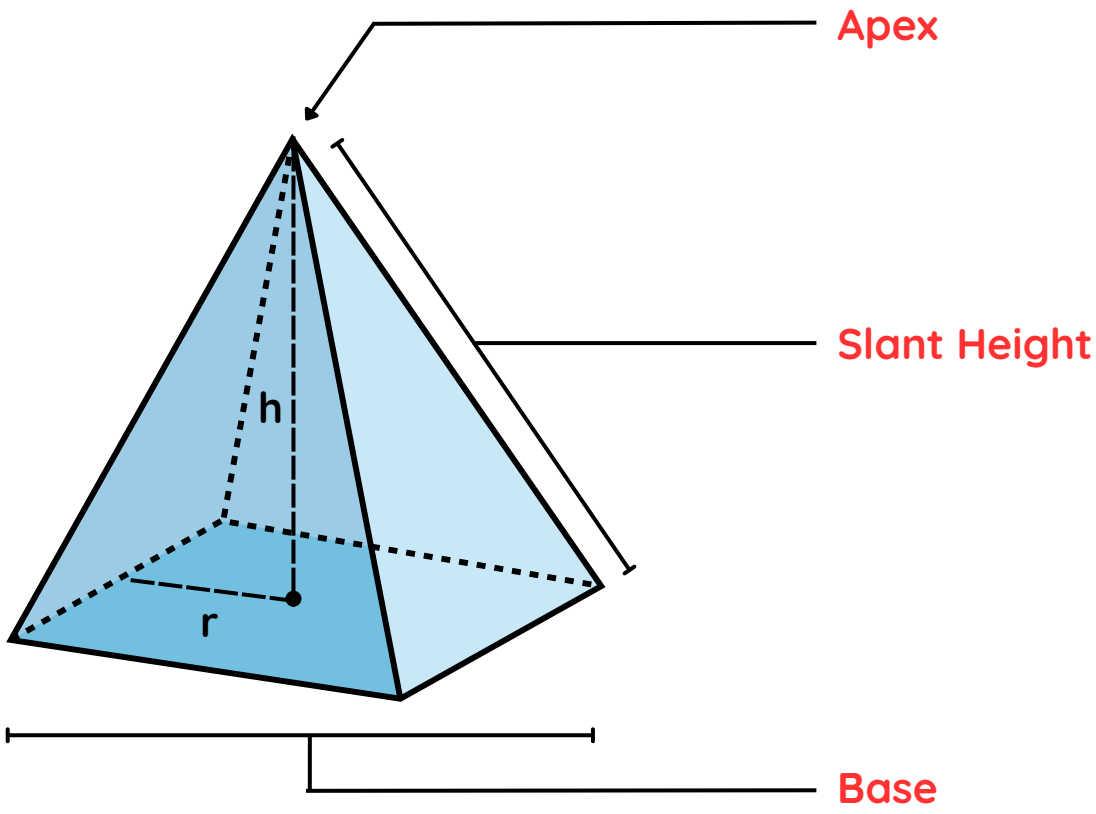
3.What is a Pyramid?

- A **Pyramid** is a three-dimensional geometric shape with a polygonal base and triangular faces that meet at a common point called the apex.

- Key Features of a Pyramid:
    - **Base –** A polygon (e.g., triangle, square, pentagon, etc.).
    - **Apex –** The topmost point where all triangular faces meet.
    - **Faces –** Triangular sides connecting the base to the apex.
    - **Edges –** The line segments where two faces meet.
    - **Height (h) –** The perpendicular distance from the base to the apex.
    - **Slant Height (l) –** The height of a triangular face from the base to the apex.



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4. How to Find the Volume of Cones and Pyramids?

- Volume is the amount of space occupied by an object.

Volume of a Cone:

- A cone has a circular base and tapers to apex.
- The formula for its volume is:

$$\text{Volume} = \frac{1}{3} \times \text{Base Area} \times \text{Vertical height}$$

$$\text{Volume} = \frac{1}{3} \times \pi r^2 \times h$$

Where,

- **r =** Radius of the base
- **h =** Height (perpendicular distance from the base to the apex)
- **π ≈** 3.1416 (pi)

Steps to Calculate Volume:

Step#1: Measure the radius (r) of the circular base.

Step#2: Measure the height (h) of the cone.

Step#3: Plug the values into the formula:

$$V = \frac{1}{3} \times \pi r^2 \times h$$

Step#4: Compute the result.

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Volume of a Pyramid:

- A **Pyramid** has a polygonal base (e.g., square, triangle) and triangular faces that meet at an apex.
- The formula for its volume is:

$$\text{Volume} = \frac{1}{3} \times \text{Base Area} \times \text{Vertical height}$$

$$\text{Volume} = \frac{1}{3} \times B \times h$$

Where,

- **B =** Area of the base
- **h =** Height (perpendicular distance from the base to the apex)

Steps to Calculate Volume:

Step#1: Find the area (B) of the base (depends on the base shape):

- **Square base: B= side^2**
- **Rectangular base: B= length × width**
- **Triangular base: B=$\frac{1}{2}$× base × height**

Step#2: Measure the height (h) of the pyramid.

Step#3: Plug the values into the formula:

$$\text{Volume} = \frac{1}{3} \times B \times h$$

Step#4: Compute the result.

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Solved Examples

Problem1: Find the volume of a cone with radius 6 cm and height 10 cm. Use **π≈3.14**

Solution:

Step#1: Find the Radius and Height of the Cone,

- r = 6cm
- h = 10cm

Step#2: Plug the Values into the formula,

$$\text{Volume} = \frac{1}{3} \times \pi r^2 \times h$$

$$\text{Volume} = \frac{1}{3} \times (3.14)\ (6^2) \times 10$$

Step#3: Compute the result,

$$\text{Volume} = \frac{1}{3} \times (3.14)\ (36) \times 10$$

$$\text{Volume} = \frac{1}{3} \times 113.04 \times 10$$

$$\text{Volume} = \frac{1}{3} \times 1130.4$$

$$\text{Volume} = 376.8\ \text{cm}^3$$

The Volume is **376.8 cm³**

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Problem2: Find the volume of a pyramid with a square base of side length 9 meters and a height of 12 meters.

Solution:

Step#1: Find the Area of the base,

Since the base is a square:

$$\text{Base Area} = 9 \times 9 = 81 \text{ m}^2$$

Step#2: Plug the Values into the formula,

$$\text{Volume} = \frac{1}{3} \times B \times h$$

$$\text{Volume} = \frac{1}{3} \times 81 \times 12$$

Step#3: Compute the result,

$$\text{Volume} = \frac{1}{3} \times 81 \times 12$$

$$\text{Volume} = \frac{1}{3} \times 972$$

$$\text{Volume} = 324 \text{ m}^3$$

The volume is **324 m³**

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5. How to Find the Surface Area of Cones and Pyramids?

- Surface area is the Total Area of all the surfaces (faces, bases, and curved sides) that cover a 3D object.

Surface area of a Cone:

- It includes:
  - Base area (a circle)
  - Lateral surface area (the curved side)
- Formula:

$$\text{Surface Area} = \text{Base Area} + \text{Lateral Area}$$

Where,

- **Base Area =** πr^2 (where r=radius)
- **Lateral Area =** $\pi r \ell$ (where ℓ=slant height)

Steps to Calculate Surface Area:

Step#1: Identify Given Values.

Step#2: Find the Base Area

Step#3: Find the Lateral (Curved) Surface Area.

Step#4: Calculate Total Surface Area.

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Surface Area of a Pyramid:

- It includes:
  - A base (which can be a square, triangle, rectangle, etc.)
  - Triangular lateral faces (number depends on the base shape)
- Formula:

$$\text{Surface Area} = \text{Base Area} + \text{Lateral Area}$$

Where,

- Lateral Area = $\frac{1}{2} \times$ Perimeter of Base $\times \ell$
- ℓ= Slant height

Steps to Calculate Surface Area:

Step#1: Identify Given Values.

Step#2: Find the Base Area

- **Square base: Area=s^2 (side length s)**
- **Triangular base: Area= $\frac{1}{2}bh$ (base b, height h)**
- **Rectangular base: Area= lw (length l, width w)**

Step#3: Find the lateral area.

Step#4: Calculate Total Surface Area.

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Solved Examples

Problem1: A cone has a radius of 5 cm and a slant height of 13 cm. Calculate its total surface area. Using **$\pi \approx 3.14$**

Solution:

Step#1: Identify Given Values,

- **Radius (r) =** 5 cm
- **Slant height (ℓ) =** 13 cm

Step#2: Find the Base Area,

The base is a circle, so its area is:

$$\text{Base Area} = \pi r^2 = \pi(5)^2 = 25\pi \text{ cm}^2$$

Step#3: Find the Lateral (Curved) Surface Area

The lateral area of a cone is given by:

$$\text{Lateral Area} = \pi r \ell = \pi(5)(13) = 65\pi \text{ cm}^2$$

Step#4: Calculate Total Surface Area,

$$\text{Surface Area} = \text{Base Area} + \text{Lateral Area}$$

$$\text{Surface Area} = 25\pi + 65\pi$$

$$\text{Surface Area} = 90\pi = 90 \times 3.14 = 282.6 \text{ cm}^2$$

The Surface Area is **282.6 cm^2**

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Problem2: A square pyramid has a base side length of 6 m and a slant height of 5 m. Find its total surface area.

Solution:

Step#1: Identify Given Values,

- **Base side length (s) =** 6m
- **Slant height (ℓ) =** 5m

Step#2: Find the Base Area,

The base is a square, so its area is:

$$\text{Base Area} = s^2 = (6)^2 = 36\ m^2$$

Step#3: Find the Lateral (Curved) Surface Area

A square pyramid has 4 triangular faces.

$$\text{Lateral Area} = \frac{1}{2} \times \text{Perimeter of Base} \times \ell$$

Where,

$$\text{Perimeter of Base} = 4s = 4 \times 6 = 24\ m$$

Now,

$$\text{Lateral Area} = \frac{1}{2} \times 24 \times 5 = 60\ m^2$$

Step#4: Calculate Total Surface Area,

$$\text{Surface Area} = \text{Base Area} + \text{Lateral Area}$$

$$\text{Surface Area} = 36 + 60 = 96\ m^2$$

The Surface Area is **96 m^2**