Answers:

Question 1:

Ans:

Domain

- This is a polynomial (no denominators or square roots to worry about).

$$\boxed{\text{Domain} = (-\infty,\, \infty).}$$

Range

- It is an "upward" parabola because the leading coefficient $2 > 0$.
- **Vertex** at $x = -\frac{b}{2a} = -\frac{-3}{2 \cdot 2} = \frac{3}{4}$.
- **Minimum value:** $f\left(\frac{3}{4}\right)$. A quick calculation shows:

$$f\left(\tfrac{3}{4}\right) = -\tfrac{1}{8}.$$

Hence the parabola's y-values go from $-\frac{1}{8}$ upwards to $+\infty$.

$$\boxed{\text{Range} = \left[-\tfrac{1}{8},\, \infty\right).}$$

Question 2:

Ans:

Domain

- Denominator $x^2 - 16 \neq 0 \Longrightarrow x \neq \pm 4.$

$$\boxed{\text{Domain} = (-\infty, -4) \cup (-4, 4) \cup (4, \infty).}$$

Range (outline)

1. **Sign analysis:**
   - For $|x| > 4$, the denominator is positive, so $f(x)$ is positive.
   - For $|x| < 4$, the denominator is negative, so $f(x)$ is negative.
2. As x approaches ± 4 from inside, $f(x) \to -\infty$; from outside, $f(x) \to +\infty$.
3. As $|x| \to \infty$, $f(x) \to 0$.

Putting it together, one finds:

- Negative values go all the way down to $-\infty$ up through $-\frac{1}{16}$.
- Positive values start just above 0 and extend to $+\infty$.

$$\boxed{\text{Range} = (-\infty,\ -\tfrac{1}{16}]\ \cup\ (0,\ +\infty).}$$

Question 3:

Ans:

Domain

- Radicand $5 - x \geq 0 \Longrightarrow x \leq 5.$

$$\boxed{\text{Domain} = (-\infty,\ 5].}$$

Range

- The square root is always ≥ 0.
- As x decreases without bound, $(5 - x)$ grows arbitrarily large, so $\sqrt{5 - x}$ can be as large as we want.

  Hence

$$\boxed{\text{Range} = [0,\ \infty).}$$

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Question 4:

Ans:

Domain

1. **Radicand** ≥ 0: $x^2 - 4 \geq 0 \Longrightarrow x \leq -2$ or $x \geq 2$.
2. **Denominator** $\neq 0$: $x + 2 \neq 0 \Longrightarrow x \neq -2$.

Combined:

$$\boxed{\text{Domain} = (-\infty,\ -2) \cup [2,\ \infty).}$$

Range (sketch)

- For $x \geq 2$, the denominator $x + 2 > 0$, while $\sqrt{x^2 - 4} \geq 0$.
  - At $x = 2$, numerator is 0, so $f(2) = 0$.
  - As $x \to \infty$, $\sqrt{x^2 - 4} \approx x$, so $\frac{\sqrt{x^2-4}}{x+2} \approx \frac{x}{x+2} \to 1$ (but never equals 1).
  - Thus in $[2, \infty)$, f runs from 0 up to (but not including) 1.
- For $x < -2$, denominator $x + 2 < 0$, numerator ≥ 0. Thus $f(x) \leq 0$.
  - As $x \to -2^-$, denominator $\to 0^-$ and numerator $\to 0^+$, so $f(x) \to -\infty$.
  - As $x \to -\infty$, $\sqrt{x^2 - 4} \approx |x|$, giving $f(x) \approx \frac{-x}{x+2} \to -1$ from below.

Hence

$$\boxed{\text{Range} = (-\infty,\ -1) \cup [0,\ 1).}$$

Question 5:

Ans:

Domain

- Polynomial, no restrictions:

$$\boxed{\text{Domain} = (-\infty,\ \infty).}$$

Range

- Write it as $x^2 - x - 2$. This is an upward-opening parabola with
  - Vertex at $x = \frac{1}{2}$.
  - Minimum $f\left(\frac{1}{2}\right) = -\frac{9}{4}$.

    Hence

$$\boxed{\text{Range} = \left[-\tfrac{9}{4},\ \infty\right).}$$

Question 6:

Ans:

Domain

- Inside the root: $-(x^2 - 6x + 8) \geq 0 \Longrightarrow x^2 - 6x + 8 \leq 0.$
- Factor: $x^2 - 6x + 8 = (x-2)(x-4)$. A quadratic with positive leading coefficient ≤ 0 *between* its roots, so

$$2 \ \leq\ x\ \leq\ 4.$$

$$\boxed{\text{Domain} = [\,2,\ 4\,].}$$

Range

- Over $[2, 4]$, the expression $-x^2 + 6x - 8$ reaches its maximum at $x = 3$ (midpoint of the roots).
  - At $x = 3$, the radicand is 1, so $f(3) = \sqrt{1} = 1$.
  - At the endpoints $x = 2$ or 4, the radicand is 0, so f is 0.

    Thus

$$\boxed{\text{Range} = [\,0,\ 1\,].}$$

Question 7:

Ans:

Domain

- Denominator $(x-1)(x+1) \neq 0$, so $x \neq \pm 1$.

$$\boxed{\text{Domain} = (-\infty, -1) \cup (-1, 1) \cup (1, \infty).}$$

Range (brief method)

1. Let $y = \frac{3}{(x-1)(x+1)}$.
2. Multiply across: $y(x^2 - 1) = 3 \implies x^2 = \frac{3}{y} + 1$.
3. We need $x^2 \geq 0$, so $\frac{3}{y} + 1 \geq 0 \implies \frac{3}{y} \geq -1$.
   - If $y > 0$, then $\frac{3}{y}$ is positive, so it is automatically ≥ -1. Hence **all positive** y occur.
   - If $y < 0$, multiplying $\frac{3}{y} \geq -1$ by y (which is negative) reverses the inequality: $3 \leq -y \implies y \leq -3$.

Also $y = 0$ never happens (that would force $3 = 0$).

Hence

$$\boxed{\text{Range} = (-\infty, \ -3] \cup (0, \ \infty).}$$

Question 8:

Ans:

Domain

- Factor the radicand: $3x - x^2 = x(3 - x)$. We need $x(3 - x) \geq 0$.
  That holds for $0 \leq x \leq 3$.

$$\boxed{\text{Domain} = [\,0, \ 3\,].}$$

Range

- Rewrite $3x - x^2 = -(x^2 - 3x)$. Its maximum (for $0 \leq x \leq 3$) occurs at $x = \frac{3}{2}$.
  - There the radicand is $\frac{9}{4}$, so $f\left(\frac{3}{2}\right) = \frac{3}{2}$.
  - At $x = 0$ or 3, the radicand is 0.
    Hence

$$\boxed{\text{Range} = [\,0, \ \tfrac{3}{2}\,].}$$

Question 9:

Ans:

Domain

- Denominator $x^2 - 9 \neq 0 \implies x \neq \pm 3$.

$$\boxed{\text{Domain} = (-\infty, -3) \cup (-3, 3) \cup (3, \infty).}$$

Range (key idea: discriminant)

1. Write $y = \frac{x+4}{x^2-9}$. Then

$$y(x^2 - 9) = x + 4 \quad \implies \quad yx^2 - x - (9y + 4) = 0.$$

2. For this quadratic in x to have real solutions, its discriminant must be ≥ 0:

$$\Delta = (-1)^2 - 4\,(y)\,[-(9y+4)] = 1 + 4y(9y+4) = 36y^2 + 16y + 1.$$

3. The expression $36y^2 + 16y + 1$ is a positive-leading-coefficient parabola in y. Its roots are

$$y = \frac{-16 \pm \sqrt{16^2 - 4 \cdot 36 \cdot 1}}{2 \cdot 36} = \frac{-16 \pm 4\sqrt{7}}{72} = \frac{-4 \pm \sqrt{7}}{18}.$$

Since it opens upward, it is ≥ 0 for $y \leq \dfrac{-4-\sqrt{7}}{18}$ or $y \geq \dfrac{-4+\sqrt{7}}{18}$.

No other obstructions appear. Therefore,

$$\boxed{\text{Range} = \left(-\infty, \tfrac{-4-\sqrt{7}}{18}\right] \cup \left[\tfrac{-4+\sqrt{7}}{18}, \infty\right).}$$

Question 10:

Ans:

Domain

- Require $4x + 1 \geq 0 \implies x \geq -\frac{1}{4}$.

$$\boxed{\text{Domain} = \left[-\tfrac{1}{4}, \infty\right).}$$

Range

- At $x = -\frac{1}{4}$, the radicand is 0, so $f\left(-\frac{1}{4}\right) = -2$.
- As $x \to \infty$, $\sqrt{4x + 1}$ grows unbounded, so $\sqrt{4x + 1} - 2 \to \infty$.

  Hence

$$\boxed{\text{Range} = [-2, \infty).}$$